LESSON 6.3.3
Linking Conclusions to Physical Theory

A conclusion is more than a description of a graph. Connect the observed pattern to a physical prediction, quantify the comparison, and state the conditions under which it is supported.

EXAMPLE: A LINEAR RESTORING FORCE PREDICTS SHM

$$F = -kx \qquad a = -\frac{k}{m}x$$

Here x is displacement from equilibrium and k is the spring constant. Acceleration opposes displacement and is proportional to it.

$$T^2 = \frac{4\pi^2}{k}m$$

Prediction: T^2 is proportional to total oscillating mass m.

This simple model assumes a light spring, a fixed support, weak damping and motion within the linear elastic range. Use the total suspended mass, including the hanger.

FOLLOW THE EVIDENCE CHAIN

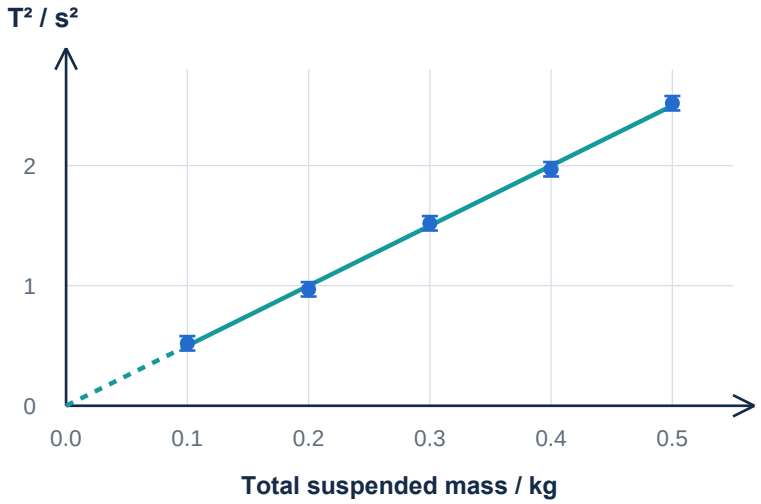
1. PREDICTION	Specify the variables, graph shape, gradient and intercept expected from the equation.
2. COMPARISON	Use measured values, units and uncertainties to judge agreement with that prediction.
3. JUDGEMENT	State support, disagreement or an inconclusive result, limited to the tested conditions.

Physics reference: OpenStax University Physics 1, §15.1 Simple harmonic motion.

1 Translate the fitted line into physics

Fit $T^2 = Sm + b$, where S is the gradient and b is the intercept. For the ideal model, $S = 4\pi^2/k$ and $b = 0$. Do not force the intercept to zero before testing it.

ILLUSTRATIVE DATA: FIVE MASSES, NOT A REAL EXPERIMENT



Blue: T^2 values 0.52, 0.97, 1.52, 1.97, 2.52 s^2, in mass order.

Bars: ±0.06 s^2. Mass uncertainty is neglected here. Teal: best-fit line.

STATED FIT UNCERTAINTIES FOR THIS WORKED EXAMPLE

$$S = (5.0 \pm 0.2)\ s^2\ kg^{-1} \qquad b = (0.00 \pm 0.06)\ s^2$$

The line above has gradient 5.0 and intercept 0.00. The parameter uncertainties are supplied estimates for the example, not regression confidence intervals calculated from these five points.

EXTRACT THE SPRING CONSTANT

$$k = \frac{4\pi^2}{S} = \frac{4\pi^2}{5.0} = 7.90\ N\ m^{-1}$$

$$\frac{\delta k}{k} \approx \frac{\delta S}{S} = 0.04 \qquad k \approx (7.9 \pm 0.3)\ N/m$$

The gradient has units s^2/kg, so its reciprocal has units kg/s^2, equivalent to N/m. The uncertainty is about 4%, giving $\delta k \approx 0.32$ N/m before rounding.

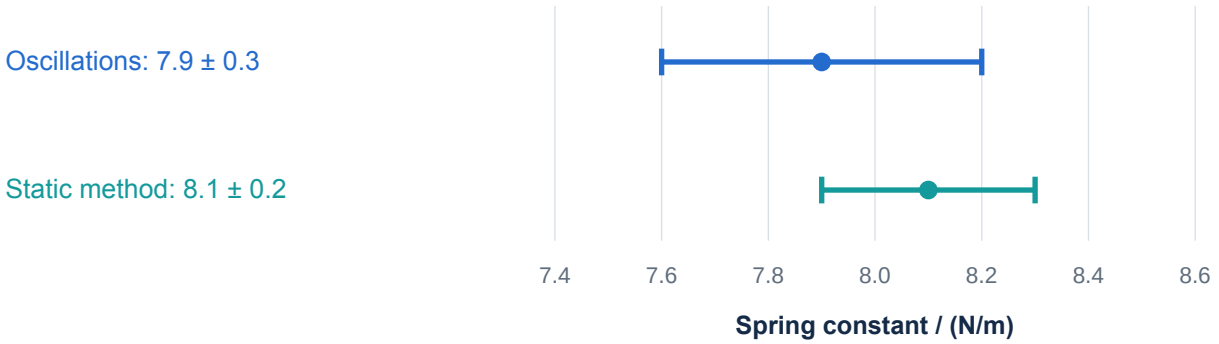
A straight line with a nonzero intercept is linear but not directly proportional. Here the intercept interval includes zero, so direct proportionality remains consistent with the data.

Analysis reference: Pearson IAL Physics specification, Unit 6 and Appendix 10.

2 Compare differences with uncertainty

Suppose an independent static measurement of the same spring gives k = (8.1 ± 0.2) N/m. Use comparable units and uncertainty estimates; the central values need not be identical.

THE STATED INTERVALS OVERLAP



The intervals 7.6-8.2 N/m and 7.9-8.3 N/m overlap. Their common values are 7.9-8.2 N/m. This supports agreement at the stated uncertainty level; it is not a statistical proof of equality.

AGREEMENT

Compatible values and an appropriate graph shape support the model within the tested range. Shared calibration errors could still hide a bias.

DISAGREEMENT

If the independent value were 9.0 ± 0.2 N/m instead, the intervals would not overlap. Investigate calibration, definitions of mass and model assumptions before blaming the theory.

INCONCLUSIVE

Large uncertainties or a narrow range may allow several models. The experiment then cannot discriminate between them; improve the measurement or choose a more informative range.

The central-value difference is only about 2.5%, but percentage difference alone is not a test of agreement. Compare it with justified uncertainties and consider any shared errors.

Interval overlap is a simple IAL comparison here. More formal statistical tests require the uncertainty type and any correlation between results.

Further reading: Pearson Student Practical Guide, conclusions and uncertainties.

3 Write a conclusion with a clear boundary

Use this worked conclusion as a structure, not a memorised paragraph. Replace every value, range and limitation with evidence from the actual experiment.

1. OBSERVED RELATIONSHIP

For total suspended masses from 0.10 to 0.50 kg, T^2 is approximately linear in mass. The fitted intercept, $(0.00 \pm 0.06)\ s^2$, is consistent with zero.

2. LINK TO THE MODEL

This agrees with the prediction $T^2 = (4\pi^2/k)m$ for a light spring with a linear restoring force and weak damping. The fitted gradient gives $k = (7.9 \pm 0.3)$ N/m.

3. QUANTITATIVE CHECK

That estimate agrees, within the stated intervals, with the independent static result (8.1 ± 0.2) N/m. The graph and parameter comparison together support the model.

4. LIMIT OF THE CLAIM

Only five masses and the chosen small-amplitude conditions were tested. The results do not establish the model for very large amplitudes, a heavy spring or masses beyond the elastic range.

BEFORE CLAIMING THAT THE THEORY WORKS

Inspect residuals (measurement minus model prediction). Patterned deviations can expose curvature or a missing effect; a high R^2 alone cannot validate the physical mechanism.

If agreement fails, check units and processing, calibrations and control variables, then the assumptions of the model. Propose a targeted test, such as comparing static and dynamic measurements over a wider safe range.

Prefer “consistent with the model under these conditions” to “the theory is proved”. A different mechanism may make a similar prediction over a limited range.

Model checks: NIST/SEMATECH e-Handbook, residual diagnostics.

Unit 6 review: planning → measurements → analysis → evaluation → conclusion.