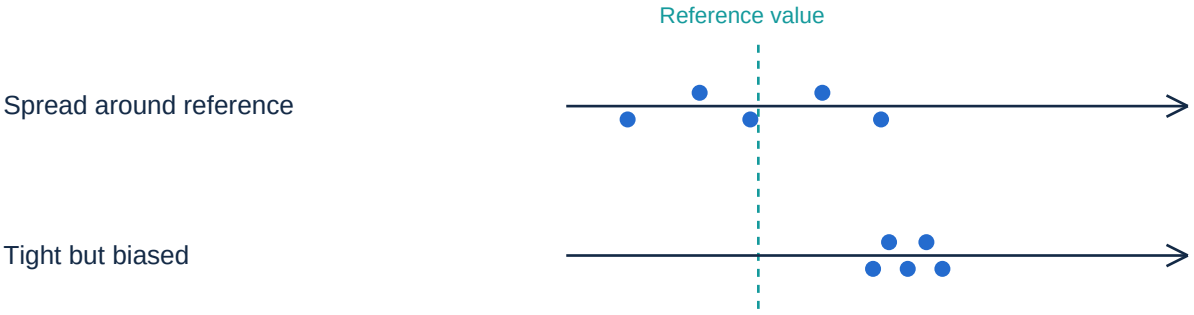
LESSON 6.2.2

Uncertainty, Calibration and Systematic Error

A tightly clustered set of readings can still be biased. Use repeated measurements to investigate scatter, and reference standards to investigate the instrument response.

PRECISION DOES NOT GUARANTEE ACCURACY



Reading increases to the right; schematic repeated measurements.

UNCERTAINTY

An estimate of the range associated with a result. It is not the actual error, which is usually unknown because the true value is unknown.

RANDOM EFFECTS

Unpredictable fluctuations produce scatter. Repeating and averaging can reduce their effect on the mean; identical displays do not imply zero uncertainty.

SYSTEMATIC EFFECTS

A consistent or predictable bias can shift results. Examples include zero offset, scale error and a method that loses energy. Repetition alone does not remove it.

State how uncertainty was estimated. A school half-range estimate is not automatically a 95% confidence interval, and a calibration correction still has uncertainty.

References: Pearson specification pp. 75-78; NIST, Basic definitions of uncertainty.

1 Estimate, then combine

Use a stated uncertainty when supplied. Otherwise apply the IAL practical conventions, considering the instrument resolution and the actual spread of readings.

A. ONE READING OR IDENTICAL REPEATS

IAL uses half the resolution as an estimate for a single reading, including a digital display. A voltmeter resolving 0.01 V gives a resolution-based estimate of ±0.005 V, not its complete accuracy specification.

B. REPEATED READINGS

Current readings: 0.294, 0.300, 0.306 A. Mean = 0.300 A.

Half range = (0.306 - 0.294)/2 = 0.006 A, or 2.0% of the mean.

IAL APPROXIMATE COMBINATION RULES

ADD / SUBTRACT	Add absolute uncertainties. For $z = x - y$, use $\delta z = \delta x + \delta y$; uncertainty magnitudes do not subtract.
MULTIPLY / DIVIDE	Add percentage uncertainties. For $z = x/y$, add the percentage uncertainties of x and y.
POWERS	Multiply percentage uncertainty by the absolute value of the power. Squaring a length doubles its percentage uncertainty.

WORKED RESISTANCE: STATED INPUT UNCERTAINTIES

V = 6.00 ± 0.06 V; I = 0.300 ± 0.006 A.

$$R = \frac{6.00}{0.300} = 20.0\ \Omega \qquad \frac{\delta R}{R} \times 100\% = 1.0\% + 2.0\% = 3.0\%$$

$\delta R = 0.030 \times 20.0 = 0.6\ \Omega$; report $R = (20.0 \pm 0.6)\ \Omega$.

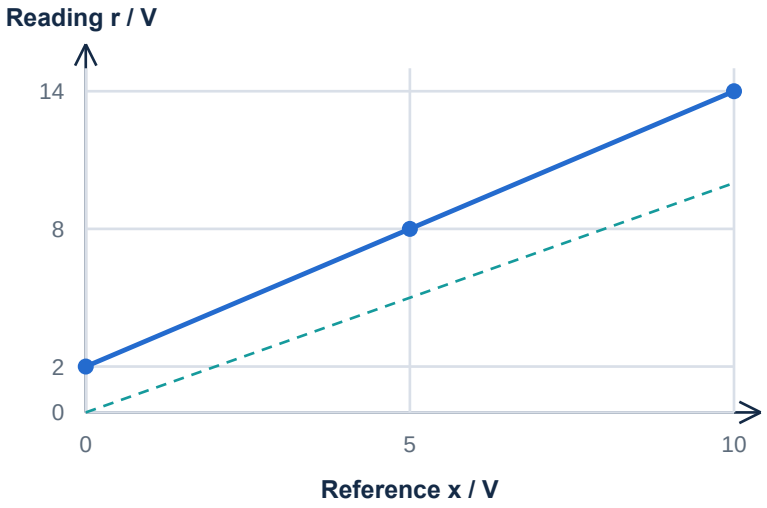
These are IAL estimates for the stated inputs, not a universal statistical rule. Shared errors can be correlated; exact constants add no uncertainty. Avoid counting the same effect twice.

Method reference: Pearson IAL Physics specification, Appendix 10, pp. 77-80.

2 Calibration: offset and scale

Compare the instrument reading r with known reference values x over the working range. A zero check alone cannot reveal every scale or nonlinearity error.

ILLUSTRATIVE VOLTMETER RESPONSE



Dashed teal: ideal response r = x.

Solid blue: r = 1.20x + 2.00 V.

A. FIT THE RESPONSE, THEN INVERT IT

$$r = ax + b \qquad x = \frac{r - b}{a}$$

The intercept b is the offset; the gradient a is the scale factor. For ideal readings in the same units, b = 0 and a = 1. Here a = 1.20, so subtracting 2.00 V alone is not enough.

WORKED CORRECTION: DISPLAY READS 11.60 V

$$x = \frac{11.60 - 2.00}{1.20} = 8.00 \text{ V}$$

B. A CORRECTION IS NOT PERFECT KNOWLEDGE

If a and b were exact, a ±0.12 V uncertainty in r would become ±0.10 V in x. In practice, include uncertainty in the reference values and fitted calibration parameters too.

Use several reference points, inspect deviations from the fit, and avoid extrapolating beyond the calibrated range. Recheck after warm-up or drift; calibration and instrument adjustment are different actions.

Reference: NIST/SEMATECH e-Handbook, §2.3 Calibration.

3 Carry uncertainty into the conclusion

Example: estimate capacitance from a discharge time constant $\tau = RC$. The uncertainties below are illustrative stated values, not the specifications of a particular instrument.

A. CAPACITANCE FROM TWO MEASURED INPUTS

$\tau = (10.0 \pm 0.4)$ s; $R = (10\,000 \pm 100)\ \Omega$.

$$C = \frac{\tau}{R} = \frac{10.0}{10\,000} = 1.00 \text{ mF}$$

$$\frac{\delta C}{C} \times 100\% = 4.0\% + 1.0\% = 5.0\%$$

$$C = (1.00 \pm 0.05) \text{ mF}$$

The interval is 0.95 to 1.05 mF. A reference value of 1.02 mF with negligible uncertainty is consistent with it; 1.12 mF is outside it. If the reference also has uncertainty, include that in the comparison.

B. SYSTEMATIC EFFECTS CAN HIDE FROM THE SCATTER

FIXED VOLTAGE SCALE FACTOR

Multiplying every positive voltage by the same factor shifts the ln V intercept, but leaves its gradient against time unchanged. The inferred time constant is unchanged in this ideal case.

VOLTAGE ZERO OFFSET

Adding an offset before taking the log generally changes the gradient and can introduce curvature. Correct a measured offset before fitting the discharge.

TIMEBASE SCALE ERROR

A timer running systematically fast or slow biases the time constant. More voltage readings cannot correct a miscalibrated time axis.

Report value, unit, uncertainty and its basis. Use sensible significant figures and match the value's decimal place to the uncertainty. Agreement supports the result; it does not prove that all errors are absent.

Further reading: Pearson Student Practical Guide; NIST uncertainty guidance.

Next: 6.2.3 Spreadsheet modelling and graphical analysis.