LESSON 6.2.1
Log Graphs
and Testing Relationships

A curved graph can hide a simple relationship. A suitable transformation makes the model linear so its gradient and intercept reveal physical constants.

1. POWER LAW: TAKE LOGS OF BOTH VARIABLES

$$y = Ax^n \quad \rightarrow \quad \ln y = n \ln x + \ln A$$

Plot ln y against ln x: gradient = n; intercept = ln A. With base-10 logs instead, the exponent n is still the gradient; the intercept is the base-10 log of A.

Examples: inverse square n = -2; direct proportion n = +1.

2. EXPONENTIAL: TAKE A LOG OF THE DEPENDENT VARIABLE

$$y = Ae^{bx} \quad \rightarrow \quad \ln y = bx + \ln A$$

Plot ln y against x: gradient = b; intercept = ln A. Positive b gives growth; negative b gives decay. Only the dependent variable is logged.

With base-10 log y against x, the gradient is b/ln 10, not b.

Notation: the compact equations above use positive numerical values in fixed units. Label physical log axes as dimensionless ratios, such as ln(V / 1 V). State both the log base and the units used to recover constants.

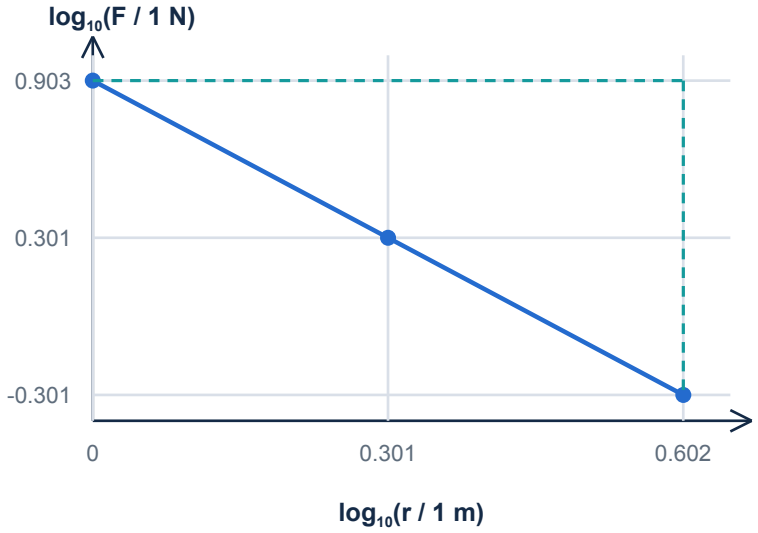
ln means base e. Use the same log base throughout. A negative log is allowed, but a zero or negative input has no real logarithm.

References: Pearson specification pp. 38-40; OpenStax Precalculus 2e, §4.5.

1 Power law: find the exponent

Illustrative force data: r = 1.0, 2.0, 4.0 m; F = 8.0, 2.0, 0.50 N. Test a power-law force model rather than judging the curve by eye.

ILLUSTRATIVE LOG-LOG DATA



Use a large gradient triangle on the fitted line, not adjacent noisy readings.

The marked points here are ideal example values, not measured results.

A. READ THE GRADIENT

$$n = \frac{-0.301 - 0.903}{0.602 - 0} = -2.00$$

Thus F follows an inverse-square law: doubling distance divides force by four. The log-log gradient is dimensionless and is unchanged if both axes use natural logs instead.

B. CONVERT THE INTERCEPT BACK

$$a = 0.903 \qquad 10^a = 8.00$$

The vertical intercept occurs where the log of (r / 1 m) is zero: r = 1 m, not r = 0. With force in N and distance in m, K = 8.00 N m².

$$F = (8.00\ \text{N m}^2)r^{-2}$$

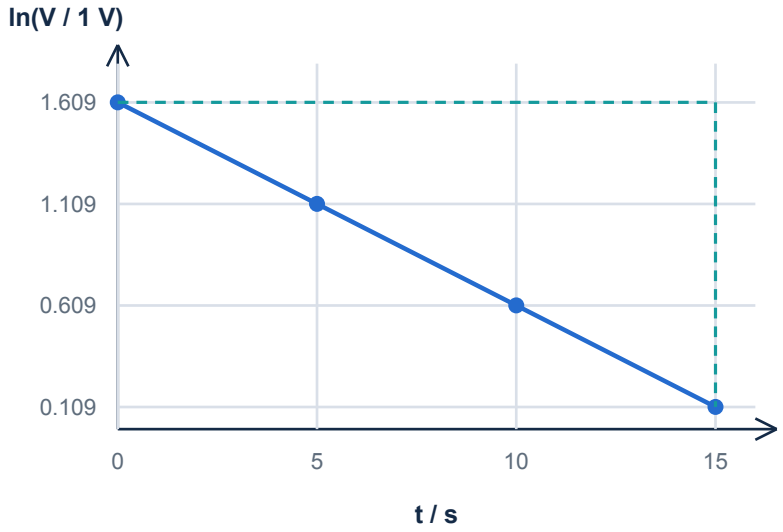
Changing units shifts the intercept; it does not change the power-law exponent.

2 Exponential decay: find the time scale

A capacitor discharges through a constant resistance. Assume negligible meter loading and use positive voltages above the measurement noise floor.

$$V = V_0 e^{-t/(RC)} \qquad \ln(V / 1\ V) = \ln(V_0 / 1\ V) - \frac{t}{RC}$$

ILLUSTRATIVE SEMI-LOG DATA



Use a large gradient triangle on the fitted line, not adjacent noisy readings.

The marked points here are ideal example values, not measured results.

A. GRADIENT AND INTERCEPT

$$s = \frac{0.109 - 1.609}{15.0 - 0} = -0.100\ s^{-1}$$

The intercept is 1.609; taking its exponential gives an initial voltage of 5.00 V. A straight ln V versus t graph supports an exponential model over the measured interval.

B. RECOVER RC AND CAPACITANCE

$$RC = -\frac{1}{s} = 10.0\ s \qquad C = \frac{10.0\ s}{10\ 000\ \Omega} = 1.00\ mF$$

Here the measured resistance is 10.0 kΩ. The time constant RC is the time to reach 1/e of the initial voltage, about 36.8%; it is not the half-life.

$$t_{1/2} = RC \ln 2 = 6.93\ s$$

If you use base-10 log V instead, multiply that gradient by ln 10 to obtain s before using RC = -1/s.

Physics reference: OpenStax University Physics 2, §10.5 RC Circuits.

3 A straight line is evidence, not proof

Test whether the slope, intercept and scatter are consistent with the proposed physical model, the measurement uncertainty and the controlled conditions.

1. CHOOSE AXES FROM THE MODEL

Power law: log-log. Exponential: log y against unlogged x. Do not try transformations until one happens to look straight and then claim the cause is established.

2. FIT THE FULL USEFUL RANGE

Use a best-fit line and a large gradient triangle. Retain extra digits in transformed values. Do not force the line through the origin unless the model and axis definitions require it.

3. CHECK THE DEVIATIONS

A residual is observed minus fitted value. Random scatter about zero is compatible with the fit; systematic curvature suggests a wrong model, an offset, changing conditions or another limitation.

4. COMPARE WITH UNCERTAINTY

Draw the steepest and shallowest acceptable lines consistent with error bars where appropriate. Report whether the predicted exponent lies within the allowed slope range.

LOGS CHANGE THE UNCERTAINTY SCALE

$$\delta(\ln y) \approx \frac{\delta y}{y} \qquad \delta(\log_{10} y) \approx \frac{\delta y}{y \ln 10}$$

For small relative uncertainties, use the approximations above. Example: $y = 2.00 \pm 0.10$ gives $\delta(\ln y) \approx 0.050$. Exact log limits are $\ln(1.90)$ and $\ln(2.10)$, which are slightly asymmetric.

BEFORE TAKING A LOG

Check zero offsets and known backgrounds. Never log zero or a negative value, or add an arbitrary constant to make it positive. Near-zero data can have very large relative uncertainty; explain any excluded range.

Curriculum alignment: Pearson IAL Physics specification, pp. 38-40.

Next: 6.2.2 Uncertainty, calibration and systematic error.