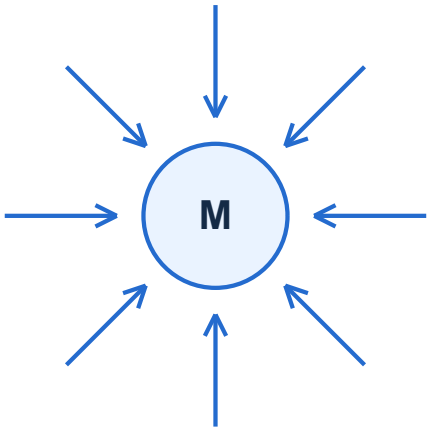
LESSON 5.4.1
Gravitational Fields and Orbital Motion

Mass creates a gravitational field. That field determines the force on a second mass, the energy needed to move it and the motion of a satellite.

A RADIAL FIELD POINTS TOWARDS THE MASS



A field is a region where a mass experiences gravitational force. Its direction is the direction of force on a small test mass.

Outside a spherical, spherically symmetric body, use its total mass at its centre. Distance r is measured from that centre.

FORCE AND FIELD STRENGTH

$$F = \frac{GMm}{r^2} \qquad g = \frac{F}{m} = \frac{GM}{r^2}$$

M is the source mass; m is the test mass. F and g above are magnitudes. The force and field vectors point inwards. Cancelling m shows that g is independent of the test mass.

$$G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$$

Field strength g has units N/kg, equivalent to m/s². G is a universal constant; g changes with the source mass and distance.

Double r: g becomes one quarter. Double M: g doubles.

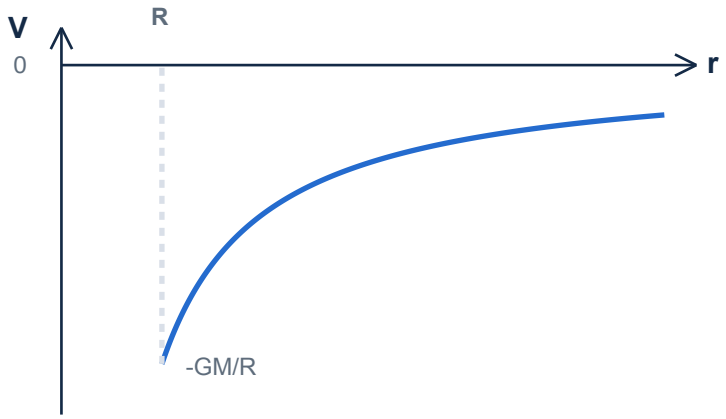
1 Potential is energy per unit mass

Gravitational potential V is the potential energy per kilogram at a point. Choose zero potential at infinite separation from the source mass.

$$V = -\frac{GM}{r} \qquad U = mV = -\frac{GMm}{r}$$

V is a scalar, measured in J/kg; U is energy in joules. A negative V means that energy must be supplied to move a mass from that point to infinity at negligible speed.

OUTSIDE A SPHERICAL BODY: r ≥ R



As r increases, V rises towards zero from below. It becomes less negative, not more negative.

The inward field points towards lower potential.

MOVING BETWEEN TWO RADII

$$\Delta U = m(V_2 - V_1) = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

For outward motion, ΔU is positive. External work equals ΔU if kinetic energy is unchanged and other energy transfers are negligible. Gravity does work -ΔU.

WORKED ENERGY CHANGE

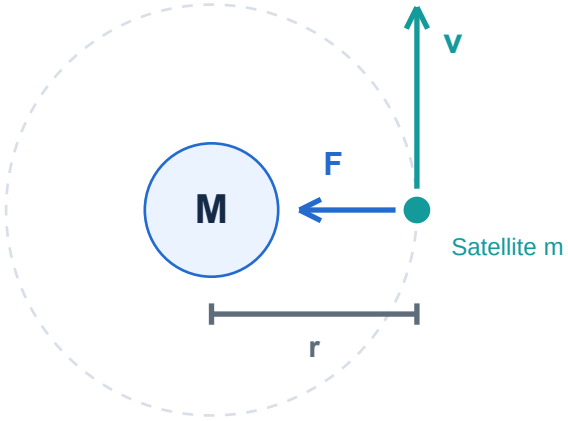
Move 2.00 kg from V = -12.0 MJ/kg to V = -8.0 MJ/kg.

$$\Delta U = 2.00 \times [-8.0 - (-12.0)] = +8.0 \text{ MJ}$$

The approximation ΔU = mgΔh applies only over a small height range where g is nearly constant; use the radial potential for large changes in distance.

2 Gravity supplies the inward force

Assume a circular orbit, negligible drag and a satellite much less massive than its central body, so the central body can be treated as fixed.



Velocity is tangent to the circle. Force and acceleration point towards the centre, changing velocity even at constant speed.

Centripetal force is the role played by gravity, not an extra force to add to it.

A. APPLY NEWTON'S SECOND LAW

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \qquad v^2 = \frac{GM}{r}$$

The satellite mass cancels. At a larger circular-orbit radius around the same body, the required speed is lower.

B. USE ONE ORBIT'S DISTANCE AND TIME

$$v = \frac{2\pi r}{T} \qquad T^2 = \frac{4\pi^2 r^3}{GM}$$

For the same central mass, T^2 is proportional to r^3. A higher circular orbit has a longer period. These constant-speed equations are not the local speed rule for an elliptical orbit.

WHY ASTRONAUTS FLOAT

The spacecraft and its occupants fall together. Gravity is still acting; the absence of a supporting contact force causes apparent weightlessness.

WHY NO CONTINUOUS THRUST?

In the ideal circular orbit, gravity is perpendicular to velocity and does no work. No thrust is needed to maintain speed when drag is negligible.

3 Apply the model and compare fields

$$r = 7.00 \times 10^6 \text{ m} \qquad GM = 3.99 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$$

Use centre-to-centre radius; if given altitude h, calculate r = R + h.

1. FIELD STRENGTH

$$g = \frac{3.99 \times 10^{14}}{(7.00 \times 10^6)^2} = 8.14 \text{ N kg}^{-1}$$

2. ORBITAL SPEED: CALCULATE v^2, THEN TAKE THE SQUARE ROOT

$$v^2 = \frac{3.99 \times 10^{14}}{7.00 \times 10^6} = 5.70 \times 10^7 \text{ m}^2 \text{ s}^{-2}$$

$$v = 7.55 \times 10^3 \text{ m s}^{-1}$$

3. PERIOD

$$T = \frac{2\pi r}{v} = 5.83 \times 10^3 \text{ s} = 97.1 \text{ min}$$

GRAVITATIONAL AND ELECTRIC FIELDS

Compare	Gravitational	Electric
Source	Mass M	Charge Q
Force	Always attractive	Attractive or repulsive
Field strength	g = F/m; N/kg	E = F/q; N/C
Point source	$g = GM/r^2$	$E = k\|Q\|/r^2$ (magnitude)
Potential	V = -GM/r	V = kQ/r (signed)
Potential units	J/kg	J/C = volt

Here k is Coulomb's constant in vacuum. Electric field direction is defined using a positive test charge. Both point-source fields are radial and obey superposition; combine fields as vectors and potentials as scalars.

Pearson IAL Physics p. 36, points 154-160: curriculum alignment.

Further reading: OpenStax University Physics 1, §§13.3 and 13.4.

Next: 5.4.2 Stars, luminosity and black-body radiation.