LESSON 5.3.3

Damping, Resonance and Forced Oscillations

An oscillator can lose energy to its surroundings, receive energy from a driver, or do both. The balance controls how its amplitude changes.

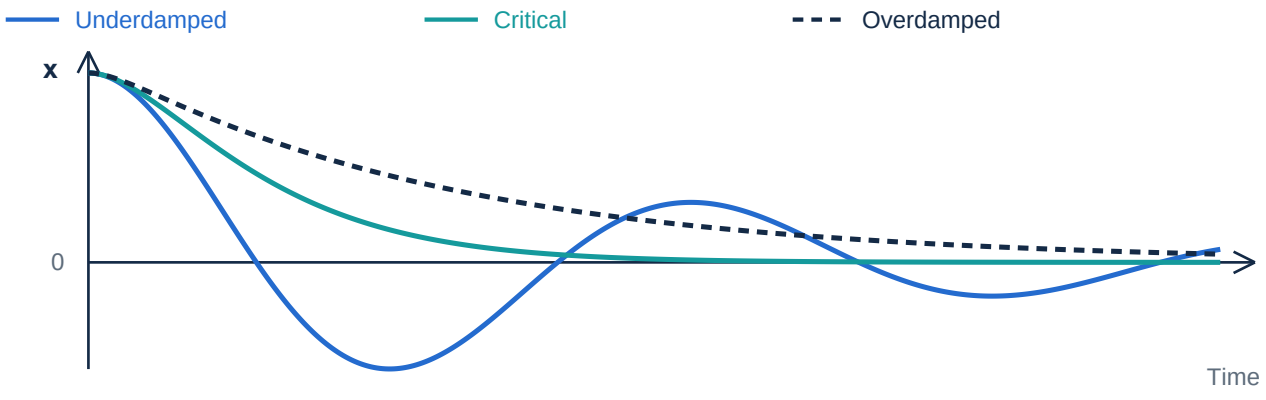
FREE OSCILLATIONS

Displace and release, with no continuing periodic drive. An ideal undamped system oscillates at its natural frequency. With light damping, the frequency is close to that value and amplitude falls.

FORCED OSCILLATIONS

A periodic external force supplies energy. After initial transients fade, a linear oscillator responds at the driving frequency, which need not equal its natural frequency.

FREE RETURN AFTER RELEASE FROM REST



1. UNDERDAMPED

Crosses equilibrium repeatedly; the amplitude decreases.

2. CRITICALLY DAMPED

Fastest return without oscillation for this release condition.

3. OVERDAMPED

No oscillation, but a slower return than critical damping.

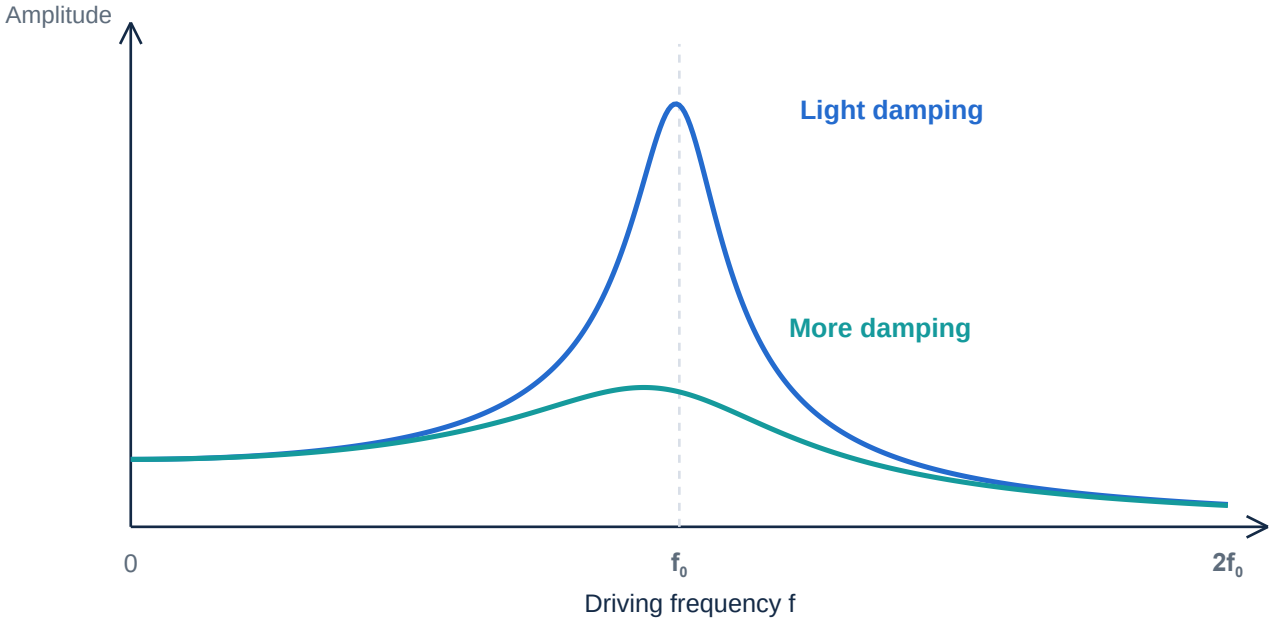
Damping transfers mechanical energy to internal energy or sound. Total energy, including the surroundings, is conserved.

1 Resonance: a large driven response

A lightly damped oscillator responds strongly when the driving frequency is close to its natural frequency. The driver transfers energy efficiently over successive cycles.

AMPLITUDE VERSUS DRIVING FREQUENCY

Same oscillator and driving-force amplitude; only damping changes.



The dashed line marks the undamped natural frequency. For displacement amplitude, damping shifts the peak slightly below this frequency. With light damping, the difference is small.

INCREASE DAMPING

The peak becomes lower and broader. The oscillator has a less sharply selected resonant frequency; a larger fraction of energy is dissipated each cycle.

STEADY AMPLITUDE

Average energy supplied per cycle equals average energy dissipated. Constant amplitude does not mean that the driver supplies no energy.

USEFUL RESPONSE OR UNWANTED VIBRATION?

A swing gains amplitude when pushes are timed to the motion. Vehicle dampers dissipate energy so the suspension settles after a bump instead of continuing to bounce.

In structures, dampers and plastic deformation of ductile components absorb mechanical energy. Plastic deformation is permanent, unlike purely elastic deformation, and can reduce vibration amplitude.

Resonance does not imply infinite amplitude: real damping and nonlinearity limit it.

2 Core Practical 16: an unknown mass

Use the same spring and a set of known masses to calibrate oscillation frequency against mass. Keep the oscillations small and damping low.

A. MEASURE KNOWN MASSES

Pull down gently and release. Time many complete free oscillations, divide by the number of cycles to obtain T, and repeat. Then f = 1/T.

B. BUILD A CALIBRATION

Plot T^2 (equivalently $1/f^2$) against known added mass m. Fit a straight line. Keep the hanger and spring unchanged; do not force the fit through zero.

C. MEASURE THE UNKNOWN

Replace the known added mass with the unknown, measure its period in the same way, and interpolate on the calibration line.

$$T^2 = Sm + C \qquad m = \frac{T^2 - C}{S}$$

S is the slope; C is the intercept. A fixed hanger and the effective moving spring mass can give a non-zero intercept. For a light ideal spring, S is approximately $4\pi^2/k$.

ILLUSTRATIVE CALIBRATION

$$T^2 = (2.00\ s^2\ kg^{-1})m + 0.0400\ s^2$$

The unknown gives T = 0.800 s. Substitute into the fitted line:

$$m = \frac{0.800^2 - 0.0400}{2.00} = 0.300\ kg$$

This is a free-oscillation timing method for the practical: there is no continuing drive. Its measured frequency approximates the resonance frequency only when damping is small.

Use repeat spread and the fit to estimate uncertainty. Keep a fixed timing reference and consistent cycle counting. Secure the support, avoid falling masses and stay within the spring's elastic limit.

Pearson IAL Physics p. 35 (148-153); Practical Guides, CP16.

Further reading: MIT 8.03, Chapter 2, Forced Oscillation and Resonance.

Next: 5.4.1 Gravitational fields and orbital motion.