LESSON 5.3.2
Energy Changes in Oscillators

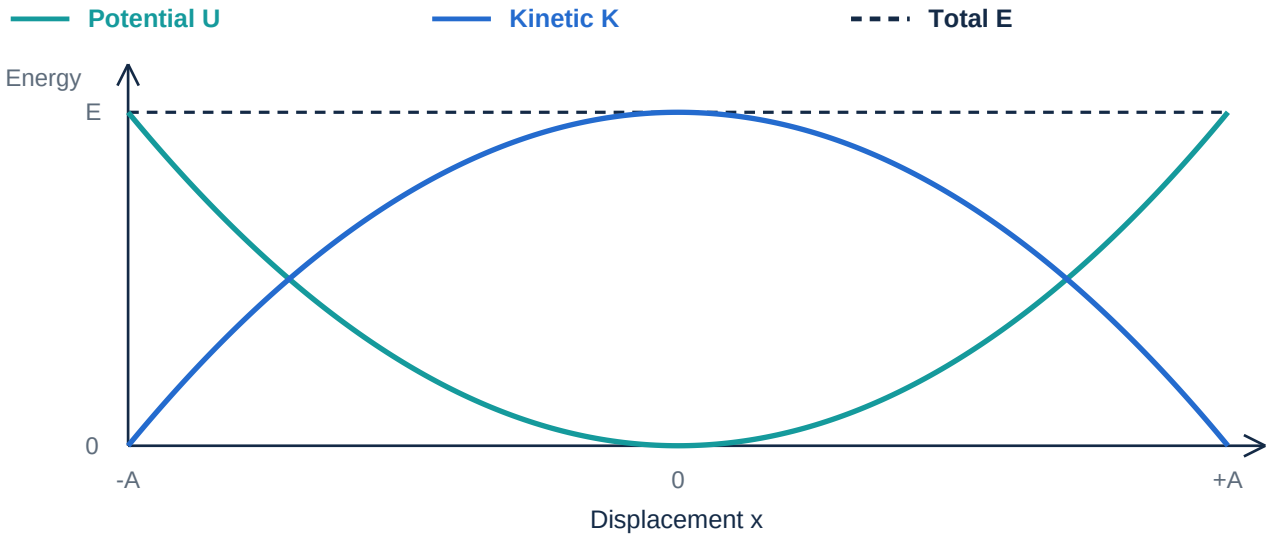
In a free, undamped oscillator, energy continually transfers between kinetic and potential stores while total mechanical energy stays constant.

IDEAL HORIZONTAL MASS-SPRING SYSTEM

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$$

x is measured from equilibrium; A is amplitude; $k = m\omega^2$; take U = 0 at x = 0.



AT THE TURNING POINTS

At x = ±A, speed is zero: K = 0 and U = E. The restoring force is not zero.

AT EQUILIBRIUM

At x = 0, U = 0 and K = E. Speed is greatest; the restoring force is zero.

1 Track the exchange, calculate the speed

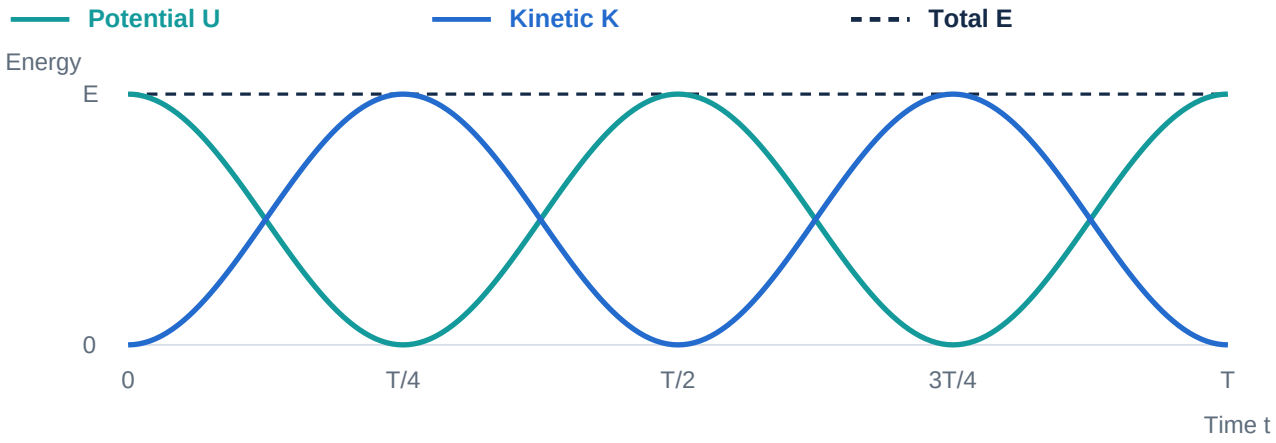
In undamped SHM, the same position has the same kinetic energy on outward and return journeys. Energy gives speed, but does not by itself give direction.

$$\frac{1}{2}mv^2 = \frac{1}{2}k(A^2 - x^2)$$

$$v^2 = \omega^2(A^2 - x^2)$$

For $x = A\cos(\omega t)$, potential and kinetic energy vary as:

$$U = E\cos^2(\omega t) \qquad K = E\sin^2(\omega t)$$



Each energy pattern repeats every T/2: twice the displacement frequency.

WORKED EXAMPLE: m = 0.400 kg, k = 40.0 N/m, A = 0.0500 m

Find the energy stores and speed at x = +0.0300 m.

$$E = \frac{1}{2} \times 40.0 \times 0.0500^2 = 0.0500\ J$$

$$U = \frac{1}{2} \times 40.0 \times 0.0300^2 = 0.0180\ J$$

$$K = E - U = 0.0320\ J$$

$$v^2 = 2K/m = 0.160\ m^2\ s^{-2} \qquad |v| = 0.400\ m\ s^{-1}$$

At this position v may be +0.400 or -0.400 m/s. The sign depends on the direction of travel; kinetic energy is non-negative in both cases.

2 Choose the system and energy zero

Mechanical energy means kinetic plus potential energy. With dissipation, mechanical energy decreases, but the total energy of the oscillator and its surroundings is conserved.

A. PENDULUM: GRAVITATIONAL POTENTIAL TO KINETIC

Choose the lowest point as zero gravitational potential energy. h is the bob's height above it; L is pivot-to-bob-centre length; θ is the angle from the downward vertical.

$$U = mgh = mgL(1 - \cos\theta)$$

At the highest point, speed is zero. At the lowest point, potential energy is least and speed is greatest. Without losses, a release from rest at height h gives:

$$\text{At the bottom:} \quad v^2 = 2gh$$

This energy argument does not require a small angle; the SHM approximation does.

B. VERTICAL SPRING: INCLUDE GRAVITATIONAL ENERGY

When x is displacement from the loaded equilibrium, the combined elastic and gravitational potential energy, relative to its equilibrium value, is $U = \frac{1}{2}kx^2$. It is not just the spring's elastic energy measured from its natural length.

An arbitrary constant in potential energy does not change the motion.

C. DAMPING: LESS AMPLITUDE MEANS LESS MECHANICAL ENERGY

Friction or drag transfers mechanical energy to internal energy and sometimes sound. Compare two turning points of the same linear oscillator: E is proportional to A^2.

$$\frac{E_2}{E_1} = \left(\frac{A_2}{A_1}\right)^2 = (0.80)^2 = 0.64$$

A 20% drop in amplitude means a 36% loss of mechanical energy, not 20%.

Source alignment: Pearson IAL Physics p. 35, points 144 and 150.

Further reading: MIT OpenCourseWare, Oscillations and Waves, Lectures 2-3.

Course lesson numbering. Next: 5.3.3 Damping, resonance and forced oscillations.