LESSON 5.3.1
Simple Harmonic Motion

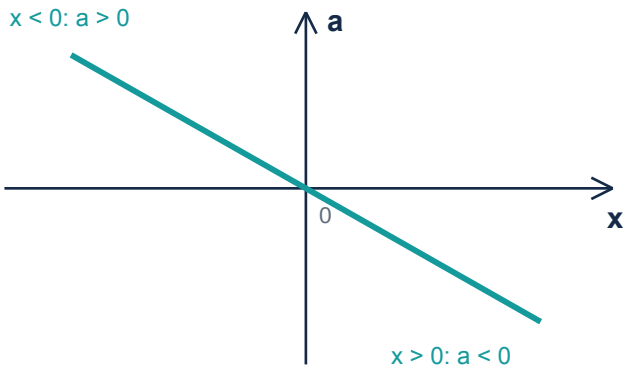
Simple harmonic motion (SHM) is an oscillation whose acceleration is proportional to displacement from equilibrium and directed towards equilibrium.

THE DEFINING CONDITION

$$F_{net} = -kx \qquad a = -\omega^2 x$$

x is signed displacement from equilibrium; k is a positive restoring-force constant. With constant mass m, Newton's second law gives $\omega^2 = k/m$. The minus sign means acceleration opposes displacement, not necessarily velocity.

ACCELERATION-DISPLACEMENT TEST



A straight line through the origin, with negative gradient, is evidence for SHM.

$$\text{gradient} = -\omega^2$$

At equilibrium: x = 0 and a = 0, but the oscillator normally passes through at its greatest speed.

AMPLITUDE A

Maximum magnitude of displacement from equilibrium, in metres. Peak-to-peak displacement is 2A.

PERIOD T AND FREQUENCY f

T is the time for one complete cycle; f is the number of cycles per second. Frequency is measured in hertz.

1 Follow one complete oscillation

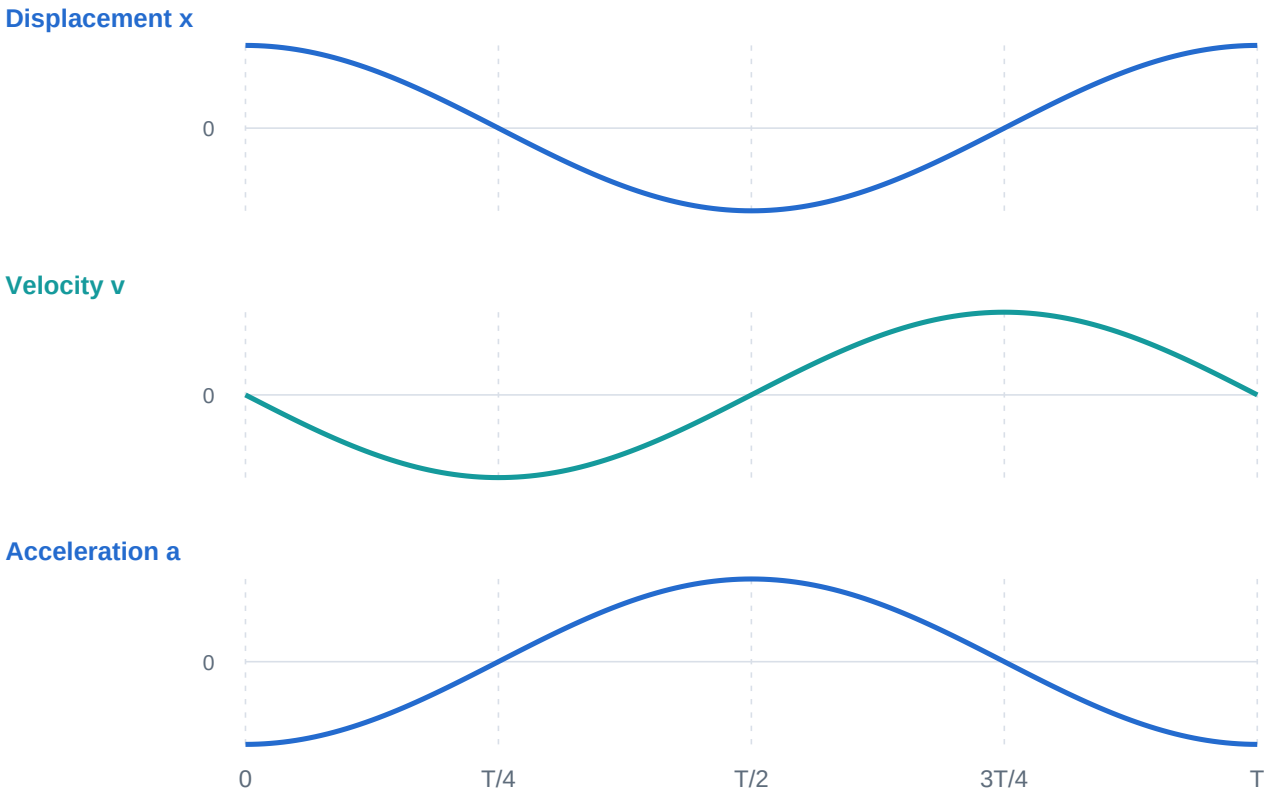
Take t = 0 at the positive turning point, so x = A and v = 0. Use radians for the phase angle in these equations.

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad f = \frac{1}{T}$$

$$x = A\cos(\omega t) \qquad v = -A\omega \sin(\omega t)$$

$$a = -A\omega^2 \cos(\omega t) = -\omega^2 x$$

Angular frequency ω is measured in radians per second, not hertz.



The gradient of x against time is v; the gradient of v against time is a. Displacement and acceleration are half a cycle out of phase. Velocity is a quarter cycle out of phase with displacement.

At turning points: speed is zero, but acceleration is greatest in magnitude. At equilibrium: acceleration is zero and speed is greatest. A negative velocity means motion in the negative direction.

For another starting state, use x = A cos(ωt + φ), with a matching phase in v and a. Choose φ from the initial position and direction of motion.

2 Apply the model, then check its limits

For ideal SHM, the period is independent of amplitude. Real systems approximate this behaviour only while their restoring force remains proportional to displacement.

MASS ON A SPRING

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Use an ideal light spring and negligible damping. For vertical motion, measure x from the loaded equilibrium, not from the natural spring length.

SIMPLE PENDULUM

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Use a small angular amplitude, a light inextensible string and a compact bob. L is pivot-to-bob-centre length; g is gravitational acceleration.

Spring: larger m gives longer T; larger k gives shorter T.

Pendulum: T depends on L and g, not bob mass; large angles need a correction.

WORKED EXAMPLE: m = 0.400 kg, k = 40.0 N/m, A = 0.0500 m

$$\omega^2 = k/m = 100\ s^{-2} \qquad \omega = 10.0\ rad\ s^{-1}$$

$$T = \frac{2\pi}{10.0} = 0.628\ s \qquad f = 1.59\ Hz$$

$$v_{max} = A\omega = 0.500\ m\ s^{-1}$$

$$|a|_{max} = A\omega^2 = 5.00\ m\ s^{-2}$$

At x = +0.0300 m, a = -3.00 m/s²: the acceleration points back towards equilibrium. The velocity may be positive or negative there, depending on which way the mass is moving.

Source alignment: Pearson IAL Physics, p. 35, points 143-147.

Further reading: MIT OpenCourseWare, Oscillations and Waves, Lectures 2-3.

Course lesson numbering. Next: 5.3.2 Energy changes in oscillators.