LESSON 5.1.3

Ideal Gas Equation

and Molecular Kinetic Energy

Connect measurable pressure, volume and temperature to the number of particles and their random motion.

$$pV = NkT$$

p	Absolute pressure, in Pa
V	Gas volume, in cubic metres
N	Number of molecules (or atoms for a monatomic gas)
T	Absolute temperature, in K

$$k = 1.38 \times 10^{-23}\ \text{J K}^{-1}\ \text{(Boltzmann constant)}$$

WORKED EXAMPLE: HOW MANY MOLECULES?

A gas occupies 2.00 litres at an absolute pressure of 100 kPa and a temperature of 300 K. First convert the units.

$$p = 1.00 \times 10^{5}\ \text{Pa}; \quad V = 2.00 \times 10^{-3}\ \text{m}^3$$

$$N = \frac{pV}{kT} = \frac{(1.00 \times 10^{5})(2.00 \times 10^{-3})}{(1.38 \times 10^{-23})(300)}$$

$$N = 4.83 \times 10^{22}\ \text{molecules}$$

Use absolute pressure, not gauge pressure: add atmospheric pressure when a gauge reports pressure above the surroundings. Convert Celsius temperatures to kelvin before substitution.

The ideal model is most useful for dilute gases, away from condensation. Particle volume and intermolecular attractions cause departures from it.

1 From collisions to kinetic energy

Use a cube of side L containing N identical molecules of mass m. Collisions are elastic and motion is equally distributed among the three directions.

1. FOLLOW THE x-COMPONENT OF ONE MOLECULE

$$\text{Impulse magnitude} = 2m|v_x|$$

$$\text{Time between hits on the same wall} = \frac{2L}{|v_x|}$$

$$\text{Mean force from one molecule} = \frac{mv_x^2}{L}$$

2. ADD THE MOLECULES AND DIVIDE BY WALL AREA

$$p = \frac{Nm\langle v_x^2\rangle}{L^3} = \frac{Nm\langle v_x^2\rangle}{V}$$

Angle brackets mean an average over molecules. Because the random motion has no preferred direction, each component contributes one third of the mean squared speed.

$$\langle v_x^2\rangle = \frac{1}{3}\langle v^2\rangle \quad \text{so} \quad pV = \frac{1}{3}Nm\langle v^2\rangle$$

3. EQUATE THIS WITH pV = NkT

$$\frac{1}{3}Nm\langle v^2\rangle = NkT$$

$$\langle E_k\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT$$

This is the mean translational kinetic energy per molecule. Here v denotes speed; the specification may use c for molecular speed. Neither symbol means the speed of light in this calculation.

2 Distinguish energy, speed and totals

At the same temperature, different ideal gases have the same mean translational kinetic energy per molecule. Lighter molecules have larger root-mean-square speeds.

WORKED EXAMPLE: ONE MOLECULE AT 300 K

$$\langle E_k \rangle = \frac{3}{2}(1.38 \times 10^{-23})(300)$$

$$\langle E_k \rangle = 6.21 \times 10^{-21}\ \text{J}$$

This is an average, not the energy of every molecule.

ROOT-MEAN-SQUARE SPEED: SQUARE, AVERAGE, ROOT

$$v_{rms}^2 = \langle v^2 \rangle = \frac{3kT}{m}$$

Take the square root of 3kT/m to find the rms speed. Do not confuse the mean of squared speeds with the square of the mean speed.

For $m = 4.65 \times 10^{-26}$ kg at 300 K:

$$v_{rms} = 517\ \text{m s}^{-1}$$

CHANGE THE TEMPERATURE

For the same gas, doubling the kelvin temperature doubles mean translational kinetic energy. The rms speed increases by a factor of 1.414, not 2.

CHANGE THE MOLECULAR MASS

At fixed temperature, a molecule with four times the mass has half the rms speed. Use mass per molecule, not the mass of the entire gas sample.

$$\text{Total translational energy} = \frac{3}{2}NkT$$

This equals internal energy for a monatomic ideal gas in the classical regime (with a consistent energy reference). Polyatomic gases can also store rotational and vibrational energy.

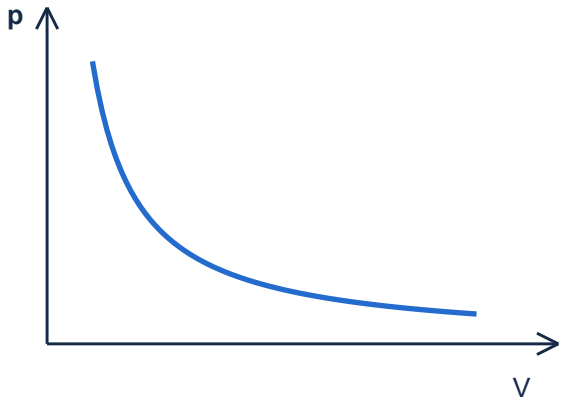
3 Core Practical 14: pressure and volume

For a fixed amount of ideal gas at constant temperature, pV is constant. Investigate this with trapped air in a syringe or Boyle-law apparatus connected to a pressure sensor.

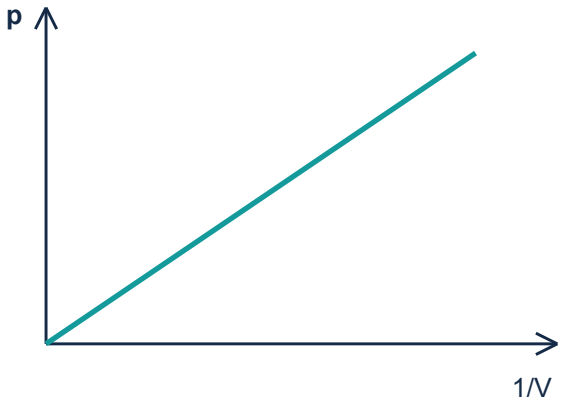
$$p_1V_1 = p_2V_2$$

Halving V doubles p only if temperature and the number of molecules stay fixed.

Inverse curve



Straight line; gradient = NkT



PLAN THE CONTROLS, NOT JUST THE READINGS

A. Keep the sample fixed.

Seal connections and check for leaks. Include trapped gas in connecting tubes in the total volume.

B. Keep the temperature fixed.

Change volume slowly and wait for thermal equilibrium before each reading. Rapid compression heats the gas.

C. Collect and analyse paired readings.

Record absolute pressure and volume over a safe range. Repeat readings; plot p against 1/V and inspect pV.

Use apparatus within its pressure rating and secure the connections. A non-zero intercept can suggest offsets or an incorrect volume correction; it is not automatically a failure of the gas law.

Source: Pearson IAL Physics, Unit 5 p. 33, points 130-132 (including CP14).

Course lesson numbering. Next: radioactive decay, half-life and activity.