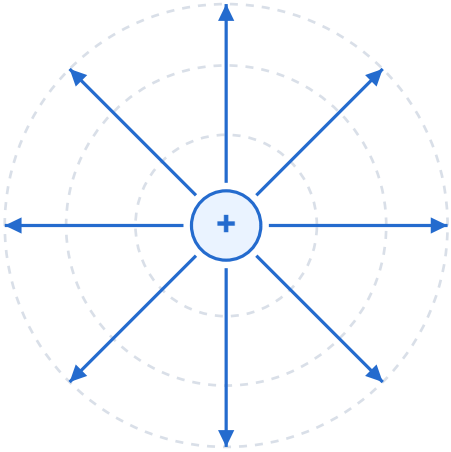
LESSON 4.2.1

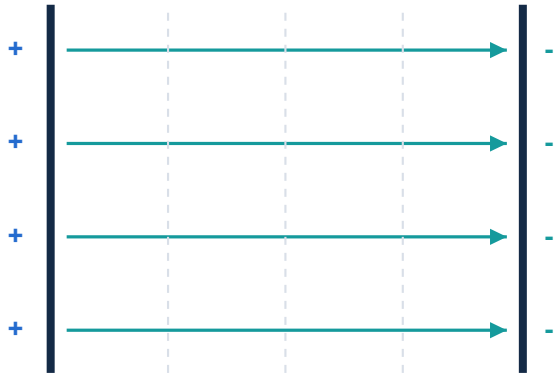
Electric Fields, Potential and Capacitance

An electric field describes the force on a charge. Electric potential describes energy per unit charge. A capacitor stores energy by separating charge.

RADIAL FIELD: POSITIVE SOURCE



UNIFORM FIELD: PARALLEL PLATES



Solid arrows: E. Dashed lines: equipotentials (cross-sections).

$$E = \frac{F}{q} \qquad F = qE \qquad |F| = \frac{k|Qq|}{r^2}$$

Field direction is the force direction on a positive test charge; a negative charge feels the opposite force. Like charges repel, unlike charges attract. Coulomb force acts along the line joining the charges.

$$k = \frac{1}{4\pi\varepsilon_0} \approx 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

Coulomb expressions apply to point charges in vacuum (approximately in air); r is their separation. Field strength has units N/C or V/m. Field lines never cross and meet equipotentials at right angles.

For a negative source, reverse radial arrows. Ignore plate-edge effects here.

1 Separate field from potential

E is a vector (force per charge). V is a scalar (potential energy per charge).

POINT CHARGE Q: ZERO POTENTIAL AT INFINITY

$$|E| = \frac{k|Q|}{r^2} \qquad V = \frac{kQ}{r} \qquad U = qV$$

Keep the sign of Q in the potential. Positive sources give positive V; negative sources give negative V. Add potentials algebraically, but add electric fields as vectors.

Doubling r quarters the field magnitude but halves the potential magnitude.

UNIFORM FIELD AND ENERGY TRANSFER

$$|E| = \frac{|\Delta V|}{d} \qquad \Delta U = q\Delta V \qquad \Delta K = -q\Delta V$$

d is separation measured along the field. Field lines point from higher to lower potential. Moving along an equipotential changes no electric potential energy.

The kinetic-energy equation assumes the electric force alone does work. For a slow move with no change in kinetic energy, external work equals the increase in potential energy.

WORKED EXAMPLE: 120 V ACROSS A 20.0 mm GAP

$$|E| = \frac{120}{0.0200} = 6.00 \times 10^3 \text{ V/m}$$

A proton moves from the higher-potential plate to the lower one: the potential change is -120 V. Its electric potential energy decreases and its kinetic energy increases:

$$\Delta K = -(1.60 \times 10^{-19})(-120) = 1.92 \times 10^{-17} \text{ J}$$

Potential is measured in volts: 1 V = 1 J/C. An electron accelerates the opposite way.

2 Charge separation stores energy

A capacitor carries +Q and -Q on its two conductors. Q means the magnitude on either conductor.

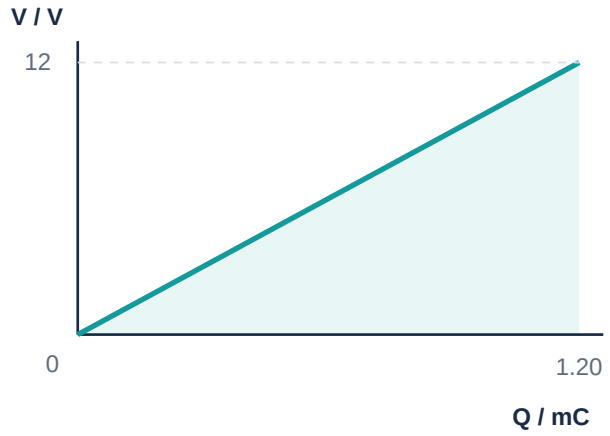
$$C = \frac{Q}{V} \qquad Q = CV \qquad 1\ F = 1\ C/V$$

Here V is the potential difference across the capacitor, and C is its capacitance. For a fixed linear capacitor, doubling V doubles Q. Use farads, not microfarads, in calculations.

$$1\ \mu F = 10^{-6}\ F \qquad 1\ nF = 10^{-9}\ F$$

The net charge on both conductors together is zero.

ENERGY = AREA UNDER THE V-Q GRAPH



As charge builds up, more work is needed for each additional coulomb. The triangular area gives stored energy, not QV.

$$W = \frac{1}{2}QV$$

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

WORKED EXAMPLE: 100 μF CHARGED TO 12.0 V

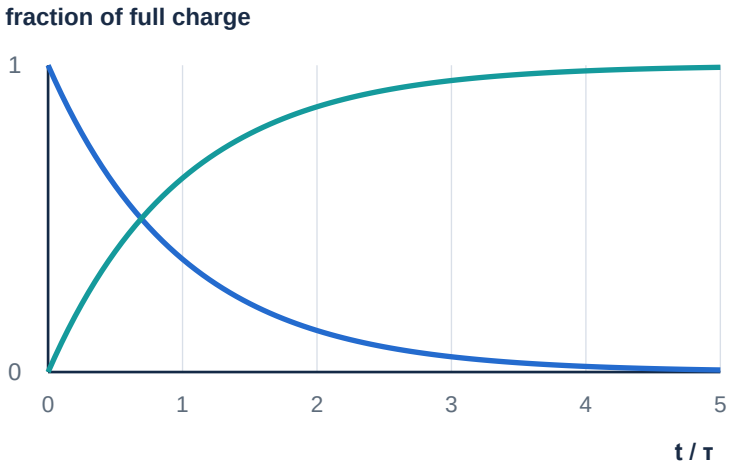
$$Q = (100 \times 10^{-6})(12.0) = 1.20 \times 10^{-3}\ C$$

$$W = \frac{1}{2}(100 \times 10^{-6})(12.0)^2 = 7.20 \times 10^{-3}\ J$$

For fixed C, doubling V quadruples stored energy. The graph uses mC, so its area is mJ.

3 Charging and discharging take time

For a resistor R and capacitor C, the time constant is $\tau = RC$, measured in seconds.



Charging

At 1τ: 63% charged

Discharging

At 1τ: 37% remains

After 5τ: about 99% charged or less than 1% remaining.

DISCHARGE THROUGH R: EXPONENTIAL DECAY

$$Q = Q_0 e^{-t/RC} \qquad V = V_0 e^{-t/RC} \qquad |I| = \frac{V_0}{R} e^{-t/RC}$$

Current magnitude decays with charge and voltage. Its sign depends on the chosen direction. A graph of ln V against t has gradient -1/RC; each additional time constant leaves 37% of the previous value.

CHARGING FROM ZERO THROUGH R WITH A CONSTANT SUPPLY

$$V_C = V_s(1 - e^{-t/RC}) \qquad Q = CV_s(1 - e^{-t/RC})$$

Charging current starts at Vs/R and decays to zero. Charge and capacitor voltage rise towards their final values, not linearly. The time constant sets the timescale.

Example: R = 100 kΩ, C = 100 μF gives τ = 10.0 s.

$$\text{After 20.0 s discharge: } V = 12.0e^{-20/10} = 1.62 \text{ V}$$

Source: Pearson IAL Physics, Unit 4 pp. 29-30, specification points 92-104.

Lesson numbering follows the Jimlaoshi course sequence. Examples are illustrative.