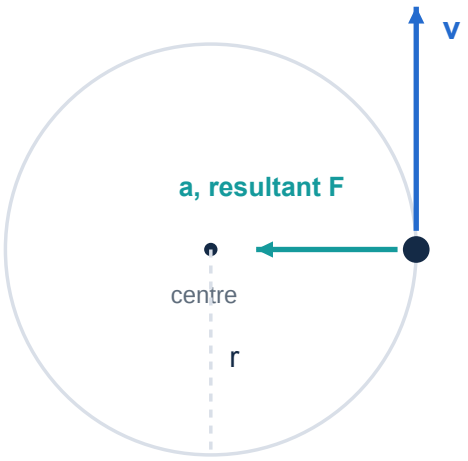
LESSON 4.1.1

Circular Motion

and Centripetal Force

Constant speed does not mean constant velocity. An object moving around a circle keeps changing direction, so it has an acceleration towards the centre.



VELOCITY: TANGENT TO THE CIRCLE

At this point the motion is upwards. The velocity is perpendicular to the radius.

ACCELERATION: TOWARDS THE CENTRE

For uniform circular motion, the resultant force is radial and inward. It turns the velocity without changing its magnitude.

CONNECT ANGLE, TIME AND SPEED

$$\theta = \frac{s}{r} \qquad 2\pi \text{ rad} = 360° \qquad 90° = \pi/2 \text{ rad}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f \qquad v = \omega r$$

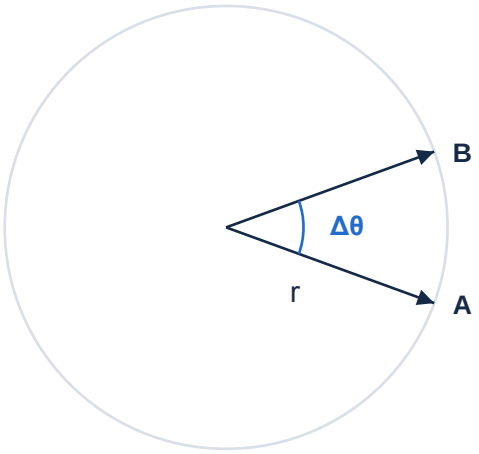
s: arc length (m); r: radius (m); θ: angle (rad); ω: angular speed (rad/s); T: time for one revolution (s); f: revolutions per second (Hz), with f = 1/T.

Use radians in angular equations. The radius is not the diameter.

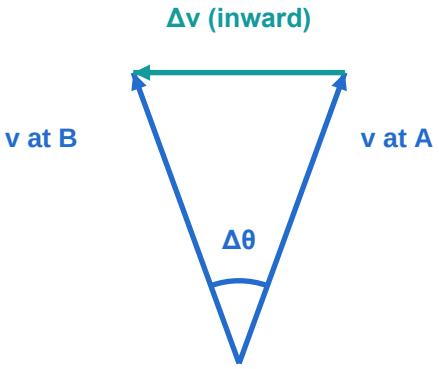
1 Why the acceleration points inwards

For uniform circular motion, two nearby velocity vectors have equal lengths but different directions.

1. RADIUS TURNS THROUGH Δθ



2. VELOCITY TURNS THROUGH Δθ



Angle enlarged for clarity; take the limit as Δθ approaches zero.

SMALL-ANGLE VECTOR RELATIONSHIP

$$|\Delta v| \approx v\Delta\theta \qquad \Delta\theta = \frac{v\Delta t}{r}$$

$$a = \lim \frac{|\Delta v|}{\Delta t} = \frac{v^2}{r} = \omega^2 r$$

The limit is $\Delta t \to 0$. Acceleration is inward, in units of m/s².

SPEED STAYS CONSTANT

In uniform circular motion the resultant force is perpendicular to velocity. It does no work, so kinetic energy stays constant.

IF SPEED ALSO CHANGES

There is also tangential acceleration. The inward component remains v^2/r, but the total acceleration is no longer purely radial.

2 Find the force that provides the turn

Centripetal force names the inward resultant, not an extra force to add to a free-body diagram.

$$F_{inward} = ma = \frac{mv^2}{r} = m\omega^2 r$$

Possible providers: tension in a string, static friction on a level road, or gravity for a circular orbit. Resolve the actual forces towards the centre and add their signed components.

WORKED EXAMPLE: PUCK ON A SMOOTH HORIZONTAL TABLE

A 0.200 kg puck moves in a circle of radius 0.500 m at 2.00 revolutions per second. A horizontal string supplies the inward force. Weight and the normal reaction balance vertically.

$$\omega = 2\pi f = 4\pi \approx 12.6 \text{ rad s}^{-1}$$

$$v = \omega r = 2\pi \approx 6.28 \text{ m s}^{-1}$$

$$a = \frac{(2\pi)^2}{0.500} \approx 79.0 \text{ m s}^{-2}$$

String tension = ma ≈ 15.8 N

STATE WHAT STAYS FIXED

At fixed mass and radius, doubling speed quadruples the force. At fixed mass and speed, doubling radius halves it. At fixed angular speed, force grows with radius.

IF THE STRING BREAKS

On the smooth table, the puck continues along the tangent at that instant. It does not move radially outwards; the inward force has disappeared.

Source: Pearson IAL Physics, Unit 4 p. 28, specification points 87-91.

Lesson numbering follows the Jimlaoshi course sequence. Example values are illustrative.