LESSON 3.2.2

Percentage Uncertainty

and Combined Uncertainty

Express uncertainty relative to the measurement, then identify how each measured input affects a calculated result.

ABSOLUTE: Δx

The uncertainty size in the same units as x. Example: ±0.004 m.

FRACTIONAL: Δx / |x|

Uncertainty divided by the magnitude of the value. It has no units.

PERCENTAGE

Fractional uncertainty multiplied by 100%. It compares relative sizes.

WORKED EXAMPLE: LENGTH

$$L = (0.800 \pm 0.004)\ \text{m}$$

$$\text{Percentage uncertainty} = \frac{0.004}{0.800} \times 100\% = 0.5\%$$

Convert both quantities to the same unit first: 4 mm / 800 mm gives the same 0.5%. The result is 0.5%, not 0.005%.

For a single reading, use the given uncertainty or the specified half-resolution estimate. For repeated data, half-range can estimate the spread. Identical repeats do not imply zero uncertainty.

CORE AND EXTENSION

Unit 3 requires percentage uncertainty but does not require compounding it. Pages 2-3 are extension material, useful for later practical analysis.

1 Match the rule to the operation

EXTENSION | Simple maximum-uncertainty estimates for separate input uncertainties.

ADD OR SUBTRACT: add absolute uncertainties

Q = a + b or Q = a - b

$$\Delta Q = \Delta a + \Delta b$$

MULTIPLY OR DIVIDE: add fractional uncertainties

Q = ab or Q = a / b

$$\frac{\Delta Q}{|Q|} \approx \frac{\Delta a}{|a|} + \frac{\Delta b}{|b|}$$

RAISE TO A POWER: multiply by the absolute power

For Q = a raised to a fixed power n

$$\frac{\Delta Q}{|Q|} \approx |n| \times \frac{\Delta a}{|a|}$$

SUBTRACTION EXAMPLE: A SMALL DIFFERENCE

$$a = (182 \pm 1)\ \text{mm} \qquad b = (150 \pm 1)\ \text{mm}$$

$$Q = 32\ \text{mm} \qquad \Delta Q = 1 + 1 = 2\ \text{mm}$$

Q = (32 ± 2) mm; percentage uncertainty = 6.25% (about 6%).

Uncertainty magnitudes are non-negative: subtracting measurements does not mean subtracting their uncertainties. These are conservative estimates, not statistical confidence intervals.

Shared errors need care: a common zero offset can cancel in a difference.

2 Carry uncertainty to the result

EXTENSION | Calculate the value first; combine relative uncertainties; convert back if needed.

1. SPEED FROM DISTANCE AND TIME

$$s = (2.40 \pm 0.02)\ \text{m} \qquad t = (1.20 \pm 0.03)\ \text{s}$$

$$v = \frac{2.40}{1.20} = 2.00\ \text{m s}^{-1}$$

$$\text{Distance uncertainty} = \frac{0.02}{2.40} \times 100\% = 0.833...\%$$

$$\text{Time uncertainty} = \frac{0.03}{1.20} \times 100\% = 2.5\%$$

$$\text{Combined percentage} = 0.833...\% + 2.5\% = 3.333...\%$$

$$\Delta v \approx 0.03333... \times 2.00 = 0.0667\ \text{m s}^{-1}$$

$$v = (2.00 \pm 0.07)\ \text{m s}^{-1}$$

2. A POWER: AREA FROM A RADIUS

$$r = (2.00 \pm 0.02)\ \text{cm} \qquad A = \pi r^2$$

Radius uncertainty = 1%; area uncertainty ≈ 2 × 1% = 2%.

$$A = (12.57 \pm 0.25)\ \text{cm}^2$$

The exact constant π contributes no measurement uncertainty.

Keep extra digits during calculation. Round the uncertainty sensibly, then give the result to the same decimal place. Improve the largest uncertainty contribution first.

Source: Pearson IAL Physics specification, Unit 3 p. 26; Appendix 10 pp. 78-80.

Lesson numbering follows Jimlaoshi; combination rules are marked as extension.