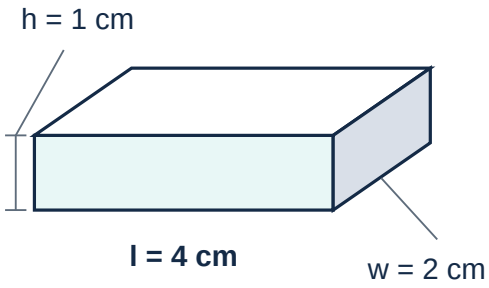
Count the surface, then divide

Surface area (SA) is the total outer area. Volume (V) is the space enclosed. SA:V compares the available exchange surface with the amount of material inside.

1. CUBOID: THREE PAIRS OF RECTANGULAR FACES



$$SA = 2(lw + lh + wh)$$

$$V = lwh$$

Include all six faces, including the bottom and the hidden faces.

2. USE THE SAME LENGTH UNIT THROUGHOUT

Convert every dimension to one unit before substitution. Work out SA and V separately. Then divide SA by V. For a cube of side a, all dimensions are equal: $SA = 6a^2$ and $V = a^3$.

3. WORKED EXAMPLE: THE CUBOID ABOVE

$SA = 2[(4 \times 2) + (4 \times 1) + (2 \times 1)] = 28\ cm^2$

$V = 4 \times 2 \times 1 = 8\ cm^3$

$SA:V = 28:8 = 3.5:1$

There are 3.5 cm^2 of surface for each 1 cm^3 of volume.

Units matter: SA/V has units of inverse length, here 3.5 cm^{-1}. The usual 3.5:1 notation compares numerical values in cm^2 and cm^3. Do not compare ratios calculated using different length units.

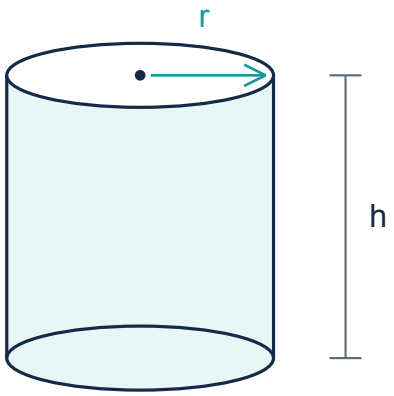
Mathematical scope: Cambridge Biology 9700, p. 63: cuboids and cylinders.
Diagrams and worked calculations are original teaching examples.

Scope: Cambridge Biology 9700 syllabus, section 4.2, outcome 3, p. 21.

A cylinder has ends as well

Model the object as a complete cylinder with both circular ends exposed. Its surface combines two circles and one curved surface.

1. LABEL RADIUS AND HEIGHT



Radius is half the diameter.

Two ends: $2\pi r^2$

Curved area: $2\pi rh$

$SA = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$

2. WORKED EXAMPLE: r = 1 cm, h = 4 cm

$SA = 2\pi(1)^2 + 2\pi(1)(4) = 10\pi \text{ cm}^2$

$V = \pi(1)^2(4) = 4\pi \text{ cm}^3$

$$SA/V = \frac{10\pi \text{ cm}^2}{4\pi \text{ cm}^3} = 2.5 \text{ cm}^{-1}$$

So SA:V = 2.5:1. Keep π until the division; it cancels exactly.

3. THREE QUICK CHECKS

Use radius, not diameter, in the formula. Include both circular ends unless the question excludes them. Do not round SA and V too early: keep full values until the final division, then give the requested precision.

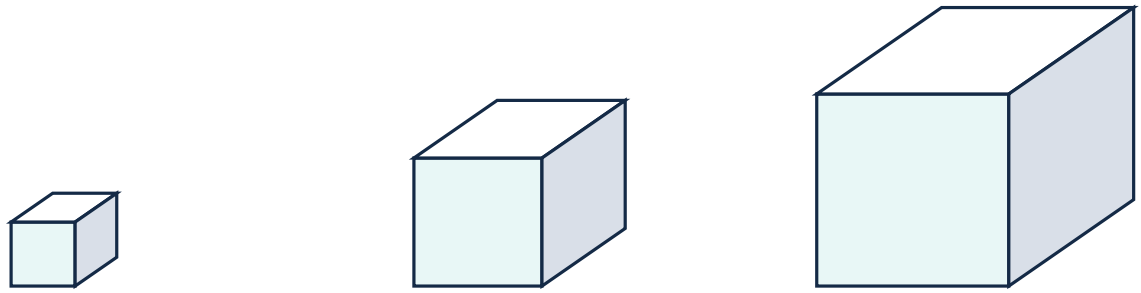
Mathematical scope: Cambridge Biology 9700, p. 63: cuboids and cylinders.
Diagrams and worked calculations are original teaching examples.

Scope: Cambridge Biology 9700 syllabus, section 4.2, outcome 3, p. 21.

Larger does not mean more per unit

For geometrically similar objects, increasing every length by the same factor increases volume faster than surface area. The total surface grows, but SA:V falls.

1. SAME SHAPE, DIFFERENT SIZES



Side = 1 mm

SA:V = 6:1

Side = 2 mm

SA:V = 3:1

Side = 3 mm

SA:V = 2:1

2. CHECK WITH NUMBERS

SIDE / mm	SA / mm²	V / mm³	SA:V
1	6	1	6:1
2	24	8	3:1
3	54	27	2:1

3. DOUBLE EVERY LENGTH: SA × 4, V × 8

The new SA/V is 4/8 of the original: it halves. More generally, multiplying all lengths by k multiplies SA by k^2 and V by k^3, so SA/V is divided by k. This rule assumes the shape stays the same.

Biological meaning: less membrane area is available per unit volume as a similarly shaped cell grows. Exchange must serve more cytoplasm, and diffusion distances to the interior are longer. Thin shapes or folded surfaces can improve exchange without simply enlarging the whole cell.

Biological context: OpenStax Biology 2e, section 4.2, Cell Size.
Scaling calculations assume the same shape and consistent length units.

Scope: Cambridge Biology 9700 syllabus, section 4.2, outcome 3, p. 21.