STATISTICS

SECTION I

Time—1 hour and 30 minutes

40 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.

1. A university that offers evening classes to a large number of students who commute to campus wants to estimate the mean commute time of all evening students. A parking facilities attendant selects one parking lot on campus, then selects the first 60 students who drive into the lot on a certain evening. Each of the 60 students is asked to provide their typical commute time to campus. The university will create a confidence interval for the mean commute time of evening students.

   Are the conditions for inference satisfied?

   (A) Yes, all conditions for inference are satisfied.

   (B) No, because the distribution of the population mean commute time is not known to be normally distributed.

   (C) No, because the sample size is greater than 10% of the population size.

   (D) No, because the students selected are not a random sample of all evening students.

   (E) No, because the sample size of 60 is not large enough to assume that the sampling distribution of the sample mean commute time is approximately normal.

2. The distribution of the length of employment for workers at a factory is moderately skewed to the right, with mean 7.4 years and standard deviation 5.6 years. The factory manager will select a random sample of 100 employees to survey.

   Which of the following best describes the sampling distribution of the sample means of length of employment for samples of size 100?

   (A) Moderately skewed to the right, with mean 7.4 years and standard deviation 5.6 years

   (B) Moderately skewed to the right, with mean 7.4 years and standard deviation less than 5.6 years

   (C) Moderately skewed to the right, with mean less than 7.4 years and standard deviation less than 5.6 years

   (D) Approximately normal, with mean less than 7.4 years and standard deviation 5.6 years

   (E) Approximately normal, with mean 7.4 years and standard deviation less than 5.6 years

3. Researchers from a national research organization investigated which platform (television, social media, newspaper, Internet, etc.) United States adults prefer to use to obtain their news. In 2016, of the 200 randomly selected United States adults that were surveyed, 114 selected television as their preferred platform. In 2018, of the 200 randomly selected United States adults that were surveyed, 98 selected television as their preferred platform. The conditions for inference are met. A 95 percent confidence interval for the difference in the population proportions of United States adults that use television as their preferred platform for obtaining news between 2016 and 2018 (2016 minus 2018) was found to be $(-0.02, 0.18)$.

   Based on the interval, is it reasonable for the researchers to claim that there is convincing statistical evidence that the population proportion of United States adults that use television as their preferred method to obtain news differed between 2016 and 2018?

   (A) No, because the researchers should have used a greater confidence level to obtain convincing evidence.

   (B) No, because zero is included in the interval there is not convincing statistical evidence that the population proportion differed between 2016 and 2018.

   (C) Yes, because zero is included in the interval there is convincing statistical evidence that the population proportion differed between 2016 and 2018.

   (D) Yes, because most of the interval is positive there is convincing statistical evidence that the population proportion increased from 2016 to 2018.

   (E) Yes, because 0.57 is greater than 0.49 there is convincing statistical evidence that the population proportion decreased from 2016 to 2018.

4. A solar power engineer took a random sample of houses and installed the same type of solar panels using two different methods on each house to investigate whether there is a mean difference in the angles of installation between the two methods for all houses in the population of interest. The engineer found the sample mean difference between the two methods to be 0.2 degree and the p-value for a two-sided matched-pairs t-test for the mean difference to be 0.65.

   Assuming the conditions for inference are met, which of the following statements is the best interpretation of the p-value?

   (A) The probability that the null hypothesis is true is 0.65.

   (B) If the null hypothesis is true, the probability is 0.65 of observing a mean difference of 0.2 degree or -0.2 degree.

   (C) If the null hypothesis is true, the probability is 0.65 of observing a mean difference of greater than 0.2 degree or less than -0.2 degree.

   (D) If the null hypothesis is true, the probability is 0.65 of observing a mean difference of greater than 0.2 degree.

   (E) If the null hypothesis is true, the probability is 0.65 of observing a mean difference of less than -0.2 degree.

5. Each accountant at a large accounting firm was classified according to accountant level (junior or senior) and method of transportation to work (walk, bus, drive alone, or carpool). The responses of the 320 accountants at the firm are summarized in the table.

	Junior	**Senior**	**Total**
Walk	25	3	28
Bus	87	12	99
Drive alone	96	25	121
Carpool	52	20	72
Total	260	60	320

What proportion of the accountants at the firm are at the senior level and carpool to work?

(A) $\frac{20}{60}$

(B) $\frac{20}{72}$

(C) $\frac{20}{320}$

(D) $\frac{112}{320}$

(E) $\frac{132}{320}$

6. The following table shows the classification of all 51 dogs from an animal shelter by whether the dogs lived mostly outdoors or mostly indoors before coming to the shelter and whether they tested positive or negative for a certain skin condition.

	Positive	**Negative**	**Total**
Mostly Outdoors	10	20	30
Mostly Indoors	8	13	21
Total	18	33	51

One dog from the animal shelter will be selected at random. Based on the table, which of the following statements is correct?

(A) The probability of selecting a dog that tested positive is greater than 0.5.

(B) The probability of selecting a dog that lived mostly indoors is greater than 0.5.

(C) The probability of selecting a dog that tested positive is greater than the probability of selecting a dog that tested negative.

(D) The probability of selecting a dog that tested positive given that the dog lived mostly outdoors is less than the probability of selecting a dog that tested positive given that the dog lived mostly indoors.

(E) The probability of selecting a dog that lived mostly outdoors given that the dog tested positive is greater than the probability of selecting a dog that lived mostly outdoors given that the dog tested negative.

7. Researchers are examining the relationship between hours of sleep and athletic performance among college athletes. Athletic performance will be measured on a numeric scale, with greater numbers indicating better performance. The researchers expect that the more hours the athletes sleep, the better they will perform. Assuming all conditions for inference are met, the researchers will create a 95 percent confidence interval for the slope of the regression line for predicting athletic performance from amount of sleep.

   For which of the following would the confidence interval support the researchers' expectations?

   (A) The confidence interval includes only positive values.
   (B) The confidence interval includes only negative values.
   (C) The confidence interval has a width less than 1.
   (D) The confidence interval has a width greater than 1.
   (E) The confidence interval includes the value 0.

8. A company that sells baby food is interested in the baby food preferences of all families with toddlers from a certain city. A representative from the company sets up a booth at one grocery store in the city that will be used to investigate baby food preference. The representative will ask people with toddlers who walk past the booth to complete the survey.

   Which of the following best describes the sampling method the company will use?

   (A) Systematic random sampling
   (B) Simple random sampling
   (C) Cluster sampling
   (D) Stratified random sampling
   (E) Convenience sampling

9. A large online retailer places packed and ready-to-be-shipped boxes in a line on a conveyor belt. A sample of boxes will be used to estimate the mean weight of boxes shipped on a given day. One proposed method for selecting the sample is as follows:

   - Generate a random integer from 1 to 20.
   - Use that integer to select an initial box from the first 20 boxes in line on the conveyor belt.
   - Select every 25th box that follows the initial box on the conveyor belt for the rest of the day.

   Which of the following best describes the proposed sampling method?

   (A) Stratified random sample, stratified by time of day
   (B) Cluster sample, with times of day as clusters
   (C) Cluster sample, with box weights as clusters
   (D) Systematic random sample
   (E) Convenience sample

10. Each voter from a random sample of 334 registered voters was asked their impression of two candidates running for the same national office. The table summarizes the responses.

	Candidate A	**Candidate B**
Favorable	138	200
Unfavorable	44	47
No opinion	88	47
Have not heard of	64	40

Which of the following is the most appropriate method to use to estimate the proportion of all registered voters who have a favorable impression of Candidate A?

(A) A one-sample z-interval for estimating a sample proportion

(B) A one-sample z-interval for estimating a population proportion

(C) A matched-pairs t-interval for estimating a mean difference

(D) A two-sample z-interval for estimating a difference between sample proportions

(E) A two-sample z-interval for estimating a difference between population proportions

11. A raffle for a charity fund-raiser is being planned. Each of 2000 raffle tickets will be sold for \$1.00. The holders of 32 winning tickets will each win a prize. The table shows the prize values and the number of prizes for each value.

Prize Value	**Number of Prizes**
\$25	20
\$50	10
\$300	2

The random variable W represents the value of the prize won for a single ticket minus the cost of the ticket. What is the expected value of W?

(A) $-\$1.00$

(B) $-\$0.20$

(C) \$0.80

(D) \$49.00

(E) \$124.00

12. In a study researching how donating to charity can affect a person's happiness, 96 participants were given \$5 a day for one week. Each participant was randomly assigned to one of two groups. Those assigned to the first group were asked to spend the money on themselves, and those assigned to the second group were asked to donate the money to charity. At the end of the week, all of the participants were asked to rate their overall level of happiness on a scale from 0 to 100, with higher scores indicating greater levels of happiness.

    Which of the following are the appropriate null and alternative hypotheses to test whether the sample data provide convincing statistical evidence that donating to charity results in greater happiness than spending on oneself, on average?

    (A) H_0: $\mu_{\text{self}} - \mu_{\text{charity}} = 0$
    H_a: $\mu_{\text{self}} - \mu_{\text{charity}} > 0$

    (B) H_0: $\mu_{\text{self}} - \mu_{\text{charity}} = 0$
    H_a: $\mu_{\text{self}} - \mu_{\text{charity}} < 0$

    (C) H_0: $\mu_{\text{self}} - \mu_{\text{charity}} = 0$
    H_a: $\mu_{\text{self}} - \mu_{\text{charity}} \neq 0$

    (D) H_0: $\mu_{\text{self}} - \mu_{\text{charity}} < 0$
    H_a: $\mu_{\text{self}} - \mu_{\text{charity}} = 0$

    (E) H_0: $\mu_{\text{self}} - \mu_{\text{charity}} > 0$
    H_a: $\mu_{\text{self}} - \mu_{\text{charity}} < 0$

13. Suppose a hypothesis test will be used to investigate whether the proportion of United States adults who have been a victim of a cybercrime, such as identity theft, is greater than 0.25. Suppose that the hypothesis test is conducted using a significance level of 0.05 and the power of the test is determined for a specific alternative value using the significance level of 0.05.

    If the significance level of the hypothesis test is changed to 0.01 and the power of the test is computed for the same alternative value using a significance level of 0.01, which of the following best describes the change(s) to the probability of a Type I error and the power of the test?

    (A) The probability of a Type I error would decrease, and the power of the test would stay the same.

    (B) The probability of a Type I error would decrease, and the power of the test would increase.

    (C) The probability of a Type I error would decrease, and the power of the test would decrease.

    (D) The probability of a Type I error would increase, and the power of the test would increase.

    (E) The probability of a Type I error would increase, and the power of the test would decrease.

14. A large company has operated assuming that the rate of absenteeism is the same for each day of the week. The new human resources director at the company claims that the assumption is false and that the rate of absenteeism is different for some days of the week as compared with others. To test the claim, a random sample of 150 absences was selected, and the day of the week on which the absence occurred was recorded. The counts are shown in the following table.

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Absences	40	27	21	34	28

The appropriate chi-square test will be performed to test the claim.

What is the contribution of the Monday absences to the calculation of the chi-square test statistic?

(A) 0.333

(B) 2.500

(C) 3.000

(D) 3.333

(E) 10.000

15. The manager of a restaurant plans to survey a random sample of customers to estimate the proportion of customers who would return within a week if given a coupon for a free dessert to use on their next visit. A similar coupon was given last year, and 30 percent of the customers who were given the coupon returned within a week. The manager will use the observed proportion from last year as a preliminary estimate of the population proportion and wants the margin of error to be at most 0.04 at a 90 percent level of confidence.

Of the following, which is the least number of customers who should be surveyed such that the estimate of the margin of error of the 90 percent confidence interval will be no greater than 0.04?

(A) 15

(B) 260

(C) 310

(D) 370

(E) 610

16. An experiment was conducted to investigate whether submersion in cold water causes a lower heart rate. The experiment used 50 volunteers. The 25 youngest volunteers had their heart rate measured while holding their breath for 30 seconds with their face submerged in cold water. The 25 oldest volunteers had their heart rate measured while holding their breath for 30 seconds with their face not submerged in cold water. The mean heart rate for volunteers who had their face submerged in cold water was lower than the mean heart rate for volunteers who did not have their face submerged in cold water.

Which of the following elements of a well-designed experiment is missing?

I. Comparison of at least two treatment groups

II. Random assignment of treatments to experimental units

III. Replication

(A) I only

(B) II only

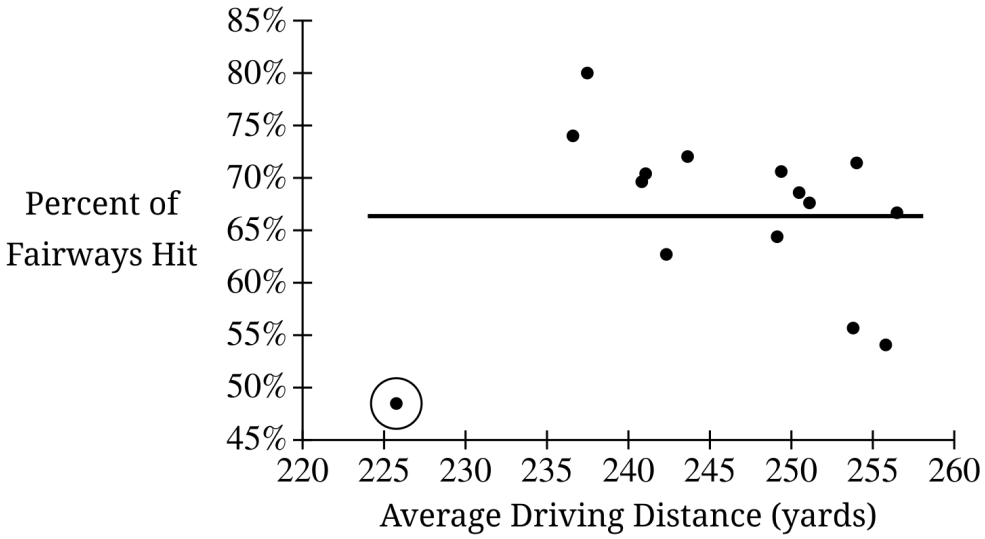
(C) I and II only

(D) II and III only

(E) I, II, and III

17. A sample of 15 golfers who played a golf course on a certain day was selected. For each golfer, the average driving distance (x), in yards, and the percent of fairways hit on the drive (y) were recorded.

The scatterplot displays the percent of fairways hit versus the average driving distance. Also shown is the least-squares regression line, $\hat{y} = 66.228 + 0.0002x$.



The point circled on the scatterplot is considered an influential point. A new least-squares regression line will be calculated with the influential point removed. How will the removal of the influential point affect the new least-squares regression line for the remaining 14 points?

(A) The y-intercept will remain the same, and the slope will be negative.

(B) The y-intercept will decrease, and the slope will be negative.

(C) The y-intercept will decrease, and the slope will be positive.

(D) The y-intercept will increase, and the slope will be negative.

(E) The y-intercept will increase, and the slope will be positive.

18. For the population of people in the United States who are at least 18 years of age, approximately 61 percent voted in a recent election. A random sample of 200 people and a random sample of 100 people will be selected from the population.

For which sample size is it more likely that greater than 70 percent of the sample voted in the recent election?

(A) They are equally likely, because the mean of the sampling distribution is the same for both sample sizes.

(B) It is more likely for a sample size of 200 people, because there is more variability in the sampling distribution for larger sample sizes.

(C) It is more likely for a sample size of 200 people, because there is less variability in the sampling distribution for larger sample sizes.

(D) It is more likely for a sample of size 100 people, because there is more variability in the sampling distribution for smaller sample sizes.

(E) It is more likely for a sample size of 100 people, because there is less variability in the sampling distribution for smaller sample sizes.

19. The probability that a particular electrical component operates successfully is $\frac{3}{4}$. An electrical system consists of three such components that operate independently. The system will function successfully if at least one of the three components operates successfully.

    What is the probability that the system will function successfully?

    (A) $\frac{1}{64}$
    (B) $\frac{27}{64}$
    (C) $\frac{37}{64}$
    (D) $\frac{3}{4}$
    (E) $\frac{63}{64}$

20. A food scientist conducting a study to investigate the nutritional habits of teenagers in a city selected a random sample of 100 teenagers in the city. Each of the 100 teenagers was asked to complete a survey with questions about age and the number of ounces of carbonated drinks consumed on the previous day.

    Assuming the conditions for inference have been met, which of the following is an appropriate test and alternative hypothesis to use to investigate whether the average number of ounces of carbonated drinks consumed in a day increases with age for teenagers in the city?

    (A) A t-test for the slope of a regression model with alternative hypothesis: H_a: $\beta > 0$
    (B) A t-test for the slope of a regression model with alternative hypothesis: H_a: $\beta < 0$
    (C) A t-test for the slope of a regression model with alternative hypothesis: H_a: $\beta = 0$
    (D) A one-sample z-test for a population proportion with alternative hypothesis: H_a: $p > 0.5$
    (E) A two-sample t-test for a difference of two population means with alternative hypothesis: H_a: $\mu_{\text{older}} \neq \mu_{\text{younger}}$

21. For a random sample of 50 newborn Holstein calves from the United States, the mean weight was 80 pounds and the standard deviation was 6 pounds. In order to compare these findings with those of other countries, an agricultural engineer converted the measured calf weights from pounds to kilograms by multiplying each weight by 0.454.

    What are the mean and standard deviation of the converted weights of the 50 newborn Holstein calves?

    (A) The mean weight is 36.32 kilograms and the standard deviation is 2.724 kilograms.
    (B) The mean weight is 36.32 kilograms and the standard deviation is 6 kilograms.
    (C) The mean weight is 80 kilograms and the standard deviation is 2.724 kilograms.
    (D) The mean weight is 80 kilograms and the standard deviation is 6 kilograms.
    (E) The mean weight is approximately 176.21 kilograms and the standard deviation is 6 kilograms.

22. The distribution of housing prices in a city includes several outliers at the upper end of the distribution and no outliers at the lower end of the distribution.

    Which of the following is the most resistant, or robust, measure of center of the distribution of housing prices?

    (A) The mean, because it is affected by the outliers.

    (B) The mean, because it is not greatly affected by the outliers.

    (C) The median, because it is affected by the outliers.

    (D) The median, because it is not greatly affected by the outliers.

    (E) The mode, because the data are categorical.

23. A study reported that 14 percent of cars registered in 2015 had a manual transmission. An automotive researcher collected data on cars registered in 2021 to investigate whether the proportion of registered cars with a manual transmission is less than 0.14. The researcher took a random sample of 150 cars registered in 2021 and found 18 of the cars had a manual transmission.

    After ensuring the conditions for inference were met, the researcher conducted an appropriate hypothesis test and calculated a p-value of 0.24. At the 5 percent level of significance, which of the following conclusions should be made?

    (A) The null hypothesis should be rejected. There is significant evidence that the population proportion of registered cars with a manual transmission in 2021 is less than 0.14.

    (B) The null hypothesis should be rejected. There is not significant evidence that the population proportion of registered cars with a manual transmission in 2021 is less than 0.14.

    (C) The null hypothesis should not be rejected. There is significant evidence that the population proportion of registered cars with a manual transmission in 2021 is not less than 0.14.

    (D) The null hypothesis should not be rejected. There is significant evidence that the population proportion of registered cars with a manual transmission in 2021 is equal to 0.14.

    (E) The null hypothesis should not be rejected. There is not significant evidence that the population proportion of registered cars with a manual transmission in 2021 is less than 0.14.

24. For which of the following distributions is the proportion of values between 900 and 1,000 the greatest?

    (A) A normal distribution with mean 900 and standard deviation 100

    (B) A normal distribution with mean 950 and standard deviation 100

    (C) A normal distribution with mean 800 and standard deviation 100

    (D) A normal distribution with mean 1,000 and standard deviation 400

    (E) A normal distribution with mean 950 and standard deviation 50

25. An owner of a novelty business operates both an online store and a retail store. The owner took a random sample of 50 sales from each store during the past year and recorded the purchase amount of each sale. Summary statistics are shown in the following table.

	Online	**Retail**
n	50	50
Mean	\$42.80	\$27.50
Standard deviation	\$10.50	\$8.25

Assuming the conditions for inference were met, the owner calculated a 95 percent confidence interval for the difference in mean purchase amounts (online minus retail) as $(\$11.55, \$19.05)$.

Which of the following is an appropriate interpretation of the 95 percent confidence interval?

(A) We are 95% confident that the purchase amount from a randomly selected online store sale will be between \$11.55 and \$19.05 greater than the purchase amount from a randomly selected retail store sale.

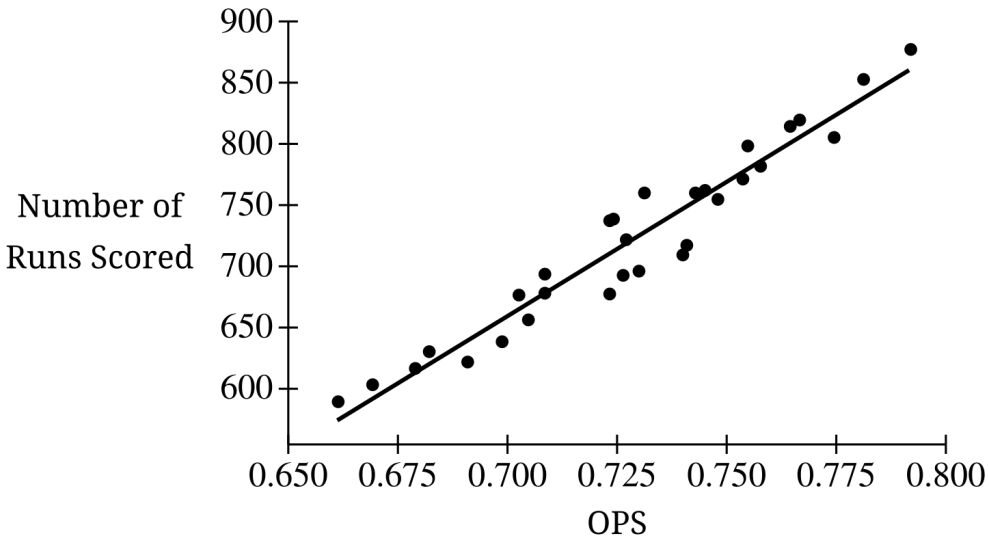
(B) We are 95% confident that the sample mean purchase amount from the online store is between \$11.55 and \$19.05 greater than the sample mean purchase amount from the retail store for the past year.

(C) We are 95% confident that the population mean purchase amount from the online store is between \$11.55 and \$19.05 greater than the population mean purchase amount from the retail store for the past year.

(D) We are 95% confident that the population mean purchase amount for the past year from both stores combined is between \$11.55 and \$19.05.

(E) We are 95% confident that if new samples of the same size were taken from each store, the sample mean purchase amount from the online store would be between \$11.55 and \$19.05 greater than the sample mean purchase amount from the retail store.

26. On-base percentage plus slugging (OPS) is a statistic used in baseball to measure a team's batting success. The number of runs scored and OPS for 30 baseball teams was used to conduct a linear regression analysis. The scatterplot and computer output for the regression analysis is shown below.



Term	Coef	SE Coef	T-Value	P-Value
Constant	−838.40	77.99	−10.75	<0.0001
OPS	2144.43	107.1	20.01	<0.0001
S= 19.516		R-Sq=93.47%		R-Sq(adj)= 93.23%

Which of the following is the most appropriate interpretation of the statistic 93.47% in the regression output?

(A) There is a strong, positive, linear relationship between number of runs scored and OPS.

(B) The typical deviation between observed and predicted number of runs scored is 0.9347.

(C) For each one-unit increase in OPS, the regression model predicts an increase of 93.47 runs scored.

(D) 93.47% of the observed number of runs scored are close to the regression line.

(E) 93.47% of the variation in number of runs scored can be explained by the linear regression with OPS.

27. A paint manufacturer conducted an experiment to investigate whether a new formula produces paint that lasts longer on the exterior of a building than paint produced using an older formula. A grid of 20 squares was drawn on one side of an exterior wall of a building. The new formula was randomly assigned to 10 squares, and the older formula was assigned to the remaining 10 squares. Each square was painted with the assigned formula. One year later, each square was rated on how well the paint had lasted, using a numerical scale for which larger values indicated longer-lasting paint.

    Which of the following best describes the design of the experiment?

    (A) A completely randomized design, because the two formulas were compared.

    (B) A completely randomized design, because each formula was randomly assigned to 10 squares.

    (C) A matched pairs design, because the same number of squares were assigned to each formula.

    (D) A matched pairs design, because two formulas were compared.

    (E) A randomized block design, because the wall was partitioned into 20 squares.

28. A 95 percent confidence interval for the proportion of parents who use parental controls for blocking, filtering, or monitoring their teenagers' online activities is (0.36,0.42).

    Which of the following could be a 99 percent confidence interval based on the same data?

    (A) $(0.30, 0.38)$

    (B) $(0.35, 0.43)$

    (C) $(0.36, 0.48)$

    (D) $(0.37, 0.41)$

    (E) $(0.40, 0.41)$

29. Luisa is a manager at a pet insurance company with many customers. Sixty percent of the company's customers filed claims in the past year. Luisa will randomly select 12 customers from all the company's customers.

    Assuming the customers selected are independent of each other, which of the following is closest to the probability that more than 2 of the 12 customers sampled did <u>not</u> file claims in the past year?

    (A) 0.003

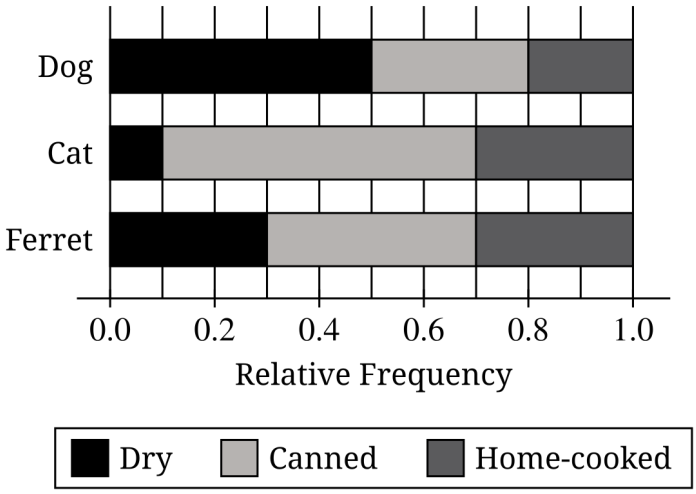
    (B) 0.400

    (C) 0.407

    (D) 0.997

    (E) 0.999

30. A veterinarian interested in pet nutrition surveyed owners of a single pet (either a dog, a cat, or a ferret) to ask whether they feed their pet primarily dry pet food, canned pet food, or home-cooked food. The following segmented bar graph shows the distribution of the three types of food for each type of pet.



Which of the following statements about pet owners in the survey is supported by the segmented bar graph?

(A) Of all owners who fed their pet primarily dry pet food, the least number owned a cat.

(B) More than half of the owners fed their pet primarily canned pet food.

(C) Ninety percent of owners fed their pets primarily dry pet food.

(D) The number of ferret owners who fed their pets primarily dry pet food was equal to the number of ferret owners who fed their pets primarily home-cooked food.

(E) A majority of dog owners fed their pets primarily canned pet food.

31. For a middle school science project, Jalen measured the pH of 25 vinegar products. The summary statistics for the values of the pH are shown in the following computer output.

Variabke	N	Mean	Median	StDev	Minimum	Maximum	Q1	Q3
pH	25	2.50	2.49	0.26	2.11	2.93	2.23	2.78

Based on the $1.5 \times$IQR rule for outliers, which of the following statements is true?

(A) The minimum pH value of 2.11 is an outlier, and no other pH value could be identified as an outlier.

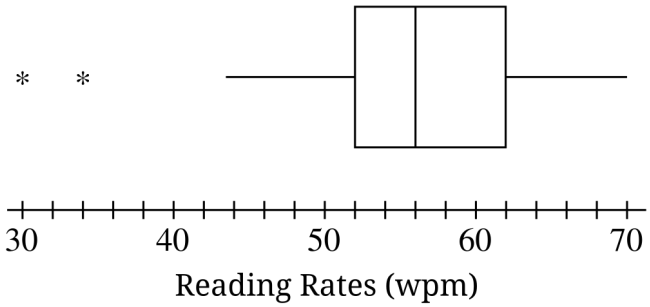
(B) The maximum pH value of 2.93 is an outlier, and no other pH value could be identified as an outlier.

(C) The minimum pH value of 2.11 is an outlier, and there could be other pH values identified as outliers that are below the first quartile, but no pH value above the third quartile could be identified as an outlier.

(D) The maximum pH value of 2.93 is an outlier, and there could be other pH values identified as outliers that are above the third quartile, but no pH value below the first quartile could be identified as an outlier.

(E) There are no pH values that could be identified as outliers.

32. The boxplot summarizes the reading rates, in words per minute (wpm), for 160 elementary school students.



Based on the boxplot, which of the following statements is <u>not</u> true for the data set?

(A) The data set contains two outliers.

(B) The range is greater than 30 wpm.

(C) At least half of the students had reading rates of at most 56 wpm.

(D) More students had reading rates between 30 wpm and 52 wpm than between 52 wpm and 70 wpm.

(E) More students had reading rates greater than 52 wpm than had reading rates less than 56 wpm.

33. A biomedical scientist from a major research university suggested that students should always ask for help when it is needed. A science teacher decided to investigate whether some students are more likely to ask for help than others. The teacher randomly selected 200 students from a Biology II course and gave them directions for a short lab procedure that were purposefully unclear. For each student, the teacher recorded whether the student asked for clarification and the grade the student earned, A, B, or C, in the preceding Biology I course. Results are listed in the following table.

	Asked for Clarification	**Did Not Ask for Clarification**	**Total**
Earned an A in Biology I	20	6	26
Earned an B in Biology I	71	31	102
Earned an C in Biology I	37	35	72
Total	128	72	200

The value of the test statistic for the appropriate chi-square test is approximately 8.25. Assume the conditions for inference were met. Do the sample data provide convincing statistical evidence of an association between grade in the preceding Biology I course and whether a student asks for clarification at the 5 percent level of significance?

(A) Yes, because the p-value is greater than 0.05.

(B) Yes, because the p-value is less than 0.05.

(C) No, because the p-value is greater than 0.05.

(D) No, because the p-value is less than 0.05.

(E) No, because the test statistic is greater than 0.05.

34. The tail length of Siberian tigers is approximately normally distributed with a mean of 0.85 meter and a standard deviation of 0.13 meter.

Which of the following is the best interpretation of the z-score for a particular Siberian tiger with a tail length of 0.8 meter?

(A) The tiger's tail length is approximately 0.38 standard deviation below the mean.

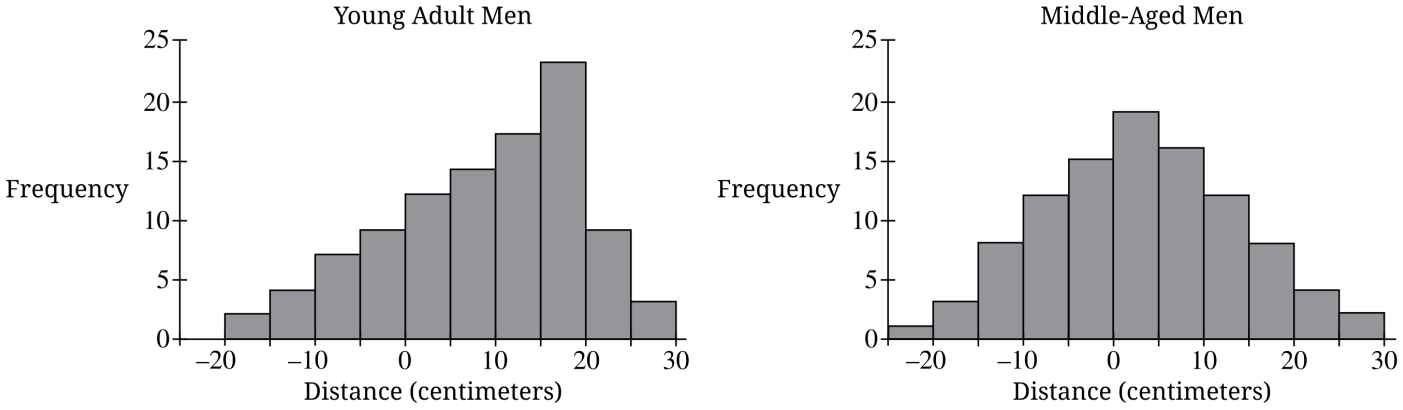
(B) The tiger's tail length is approximately 0.38 standard deviation above the mean.

(C) The tiger has an approximate 0.38 probability of having a tail length of 0.8 meter.

(D) The tiger's tail length is approximately 0.0065 meter greater than the standard deviation.

(E) The tiger's tail length is approximately 0.05 meter below the mean.

35. A common measure of flexibility is the sit-and-reach test, where a person sits on the floor with legs straight in front. The person then reaches as far forward as possible, and the distance reached past the feet is recorded. The histograms summarize the results of the sit-and-reach test for two groups, young adult men and middle-aged men. Negative distances represent a reach that is not as far as the feet.



Which of the following statements is the best description of the distributions of distances?

(A) The distribution for young adult men is approximately symmetric, and the distribution for middle-aged men is skewed to the right.

(B) The distribution for young adult men is approximately symmetric, and the distribution for middle-aged men is skewed to the left.

(C) The distribution for young adult men is skewed to the right, and the distribution for middle-aged men is approximately symmetric.

(D) The distribution for young adult men is skewed to the left, and the distribution for middle-aged men is approximately symmetric.

(E) The distribution for young adult men is skewed to the left, and the distribution for middle-aged men is skewed to the right.

36. The yields, in bushels per acre, of soybeans for 25 randomly selected counties in the midwestern region of the country in 2016 are summarized in the table.

n	$\bar{x}$	s
25	45.88	7.94

Assuming all conditions for inference are met, which of the following is a 99 percent confidence interval for the population mean yield of soybeans for all counties in the midwestern region in 2016?

(A) $45.88 + 2.797\left(\dfrac{7.94}{25}\right)$

(B) $45.88 + 2.797\left(\dfrac{7.94}{5}\right)$

(C) $45.88 + 2.576\left(\dfrac{7.94}{5}\right)$

(D) $45.88 + 2.787\sqrt{\dfrac{7.94}{25}}$

(E) $45.88 + 2.797\sqrt{\dfrac{7.94}{25}}$

37. Ming is learning the game of tennis and will practice serving a tennis ball within bounds. Assume that the probability of Ming serving a tennis ball within bounds is 0.40 and that her serves are independent of each other.

    What is the probability that the first time Ming serves a tennis ball within bounds will occur on her 4th attempt?

    (A) 0.4

    (B) $1-(0.4)^4$

    (C) $(0.6)^3(0.4)$

    (D) $\binom{4}{1}(0.4)(0.6)^3$

    (E) $\binom{4}{1}(0.4)(0.6)^3+\binom{4}{2}(0.4)^2(0.6)^2+\binom{4}{3}(0.4)^3(0.6)$

38. Caleb designed an experiment to investigate whether listening to music or chewing gum affects the time to complete a level of a certain video game. Before the game started, each person in a sample was randomly assigned to either listen to music or not listen to music. Each person was also randomly assigned to either chew gum or not chew gum. At the end of the game, the time to complete the level for each person in the sample was recorded.

    Which of the following correctly identifies the response variable of the experiment and whether the variable is categorical or quantitative?

    (A) The response variable is the time to complete the level and the variable is quantitative.

    (B) The response variable is the time to complete the level and the variable is categorical.

    (C) The response variable is whether the person was assigned to listen to music or chew gum and the variable is categorical.

    (D) The response variable is whether listening to music or chewing gum affects time to complete the level and the variable is quantitative.

    (E) The response variable is whether listening to music or chewing gum affects time to complete the level and the variable is categorical.

39. For a particular video game, Ariana's mean number of points earned per game is 190, and the standard deviation is 40. Suppose that she plays 50 games in one evening, and her goal is to obtain a mean score of at least 200 for the 50 games.

    How many standard deviations above 190 is her goal?

    (A) 0.25

    (B) 1.41

    (C) 1.77

    (D) 11.18

    (E) 12.50

40. The distribution of weight of a small watermelon is known to have a mean of 5 pounds and a standard deviation of 1.2 pounds. Six small watermelons fit in a crate, and the crate weighs exactly 1 pound.

Assuming the weights of small watermelons are independent, what is the standard deviation of the total weight of a random sample of 6 small watermelons and the crate?

(A) $\sqrt{6^2(1.2)^2}$

(B) $1+\sqrt{6^2(1.2)^2}$

(C) $1+6^2(1.2)^2$

(D) $\sqrt{6(1.2)^2}$

(E) $1+\sqrt{6(1.2)^2}$

END OF SECTION I

STATISTICS

SECTION II

Time—1 hour and 30 minutes

6 Questions

Part A

Suggested Time — 1 hour and 5 minutes

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. A random sample of 25 cans of soda was selected from a factory that produces soda. The amount of soda, in milliliters (ml), contained in each sampled can was measured. The data are shown in the following stem-and-leaf plot.

34	7					
35	1	2	3	3	4	4
35	5	5	5	6	7	9
36	2					
36	5	6	6	7	8	8
37	0	3	3	4		
37	5					

Key: 34 | 7 represents 347 ml of soda

(a) Describe the distribution of amount of soda in a can.

The factory uses two machines to fill the soda cans: Machine A and Machine B. The following back-to-back stem-and-leaf plot shows the soda amounts from the same random sample categorized by machine.

Machine A							Machine B					
						34	7					
					4	35	1	2	3	3	4	
					5	35	5	5	6	7	9	
					2	36						
8	8	7	6	6	5	36						
		4	3	3	0	37						
					5	37						

Key: 34 | 7 represents 347 ml of soda

(b) Calculate the median and IQR for each machine.

- Machine A median:
- Machine A IQR:
- Machine B median:
- Machine B IQR:

(c) Compare the variability of the measurements in the amount of soda filled per can by the two machines by referencing the appropriate measure calculated in part (b). Justify your answer.

2. An educational researcher has noticed that because many students can type faster than they can write, lecture notes taken by students using laptops tend to look like a transcript of the instructor's lecture, while handwritten lecture notes tend to be shorter summaries of the key points made during the lecture. The researcher wants to investigate whether the difference in note-taking methods has an impact on student quiz performance and is considering the following two proposals for the investigation.

   - Proposal I: The researcher will select a random sample of students from a list of volunteers and allow the students to use their preferred note-taking method, laptops or handwritten.
   - Proposal II: The researcher will randomly assign the note-taking method (laptops or handwritten) to each student in a group of volunteers, so that half of the volunteers are assigned to each method.

   (a) Which of the two proposals is not an experiment? Justify your answer.

   (b) Could either of the two proposals be used to establish a cause-and-effect relationship? Justify your answer.

   (c) Describe how student study habits could be a confounding variable when using Proposal I for the investigation.

3. In the western United States, water is sometimes diverted from mountain streams and rivers to be used for agriculture and municipal water supplies. Diverting too much water raises the water temperature in the mountain streams and rivers, affecting the fish that live there.

   A certain mountain river is abundant in brown trout and has water diverted from it for city use. Assume that the water temperature along all locations of the river has a distribution with mean 18°C and standard deviation of 2°C.

   A sport-fishing group plans to measure the water temperature at 64 randomly selected locations along the river. Let the random variable F represent the mean water temperature, in degrees Celsius, of 64 locations along the river selected at random by the sport-fishing group.

   (a) Describe the distribution of F.

   (b) Fish, such as brown trout, become stressed once water temperatures exceed 18.3°C. Determine the probability F will exceed 18.3°C. Show your work.

   A large metropolitan water district that uses water diverted from the same river is also planning to measure the water temperature at 100 randomly selected locations along the river. Because of sampling variability, the sample collected by the water district is likely to yield a sample mean water temperature different from the sample mean water temperature collected by the sport-fishing group. Let the random variable D represent the mean water temperature, in degrees Celsius, of 100 locations along the river selected at random by the large metropolitan water district.

   (c) Describe the distribution of $D - F$.

4. A veterinarian is comparing two diets, Diet J and Diet W, designed to help overweight large-breed dogs lose weight. Thirty-two overweight large-breed dogs were randomly assigned to one of the two diets, with 16 dogs in each group. The dogs remained on the assigned diet for six months and the weight loss, in pounds, was recorded for each dog. Summary statistics of the weight loss for the two groups are shown.

	Mean (pounds)	**Standard Deviation (pounds)**
Diet J	7.523	0.819
Diet W	6.379	1.012

Assume the distribution of weight loss for all overweight large-breed dogs who use Diet J is approximately normal and assume the distribution of weight loss for all overweight large-breed dogs who use Diet W is approximately normal. Do the data provide convincing statistical evidence, at the significance level of $\alpha = 0.05$, that Diet J is more effective than Diet W in terms of mean weight loss for overweight large-breed dogs that are similar to those used in the study? Complete the appropriate inference procedure to support your answer.

5. A survey conducted by a national research center in 2018 asked a random sample of people over age 12 years old in the United States whether it is very important to them personally to have a lot of money. Responses are summarized based on age in the following table of counts.

	Adults	**Teenagers**
Very important	281	86
Not very important	180	168
Total	461	254

(a) Construct a graph to compare the responses of adults and teenagers.

(b) A hypothesis test will be conducted to investigate whether the proportion of all United States adults who would respond that it is very important to have a lot of money is different from the proportion of all United States teenagers who would respond that it is very important to have a lot of money. Identify the appropriate test and state the hypotheses.

(c) The p-value of the hypothesis test was found to be less than 0.0001. Based on the p-value, what is the appropriate conclusion for the test? Justify your answer.

STATISTICS

SECTION II, Part B

Suggested Time — 25 minutes

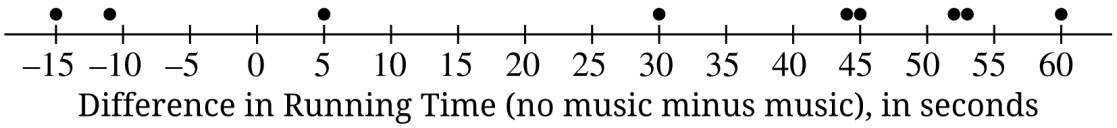
1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Nine experienced runners participated in a well-designed controlled experiment to investigate whether listening to music while running will allow a runner to run faster. On each of two consecutive Saturdays, the runners ran on a track until they completed 3.1 miles. Researchers recorded the time, in seconds, for each runner to complete

   3.1 miles. On one day the runners listened to music while running, and on the other day the runners did not listen to music. For each runner, a coin was flipped to determine on which day, the first or second, music was used.

   The following dotplot shows the differences in running times (no music minus music) for the 9 runners.



   (a) For the investigation, what does the positive difference of 60 indicate?

If listening to music has no effect on running time, then running time is equally likely to be greater for either condition, music or no music. Under that assumption, the probability that the difference (no music minus music) will be positive for any particular runner is $p = 0.5$. Let X represent the number of runners out of 9 runners who have a positive difference (no music minus music) in running time.

(b) Assume that the differences in running times are independent of one another. From the sample of 9 runners, 7 have a positive difference (no music minus music).

i. If listening to music has no effect on running time, what is the probability that among 9 runners, 7 or more runners will have a positive difference in running times? Show your work.

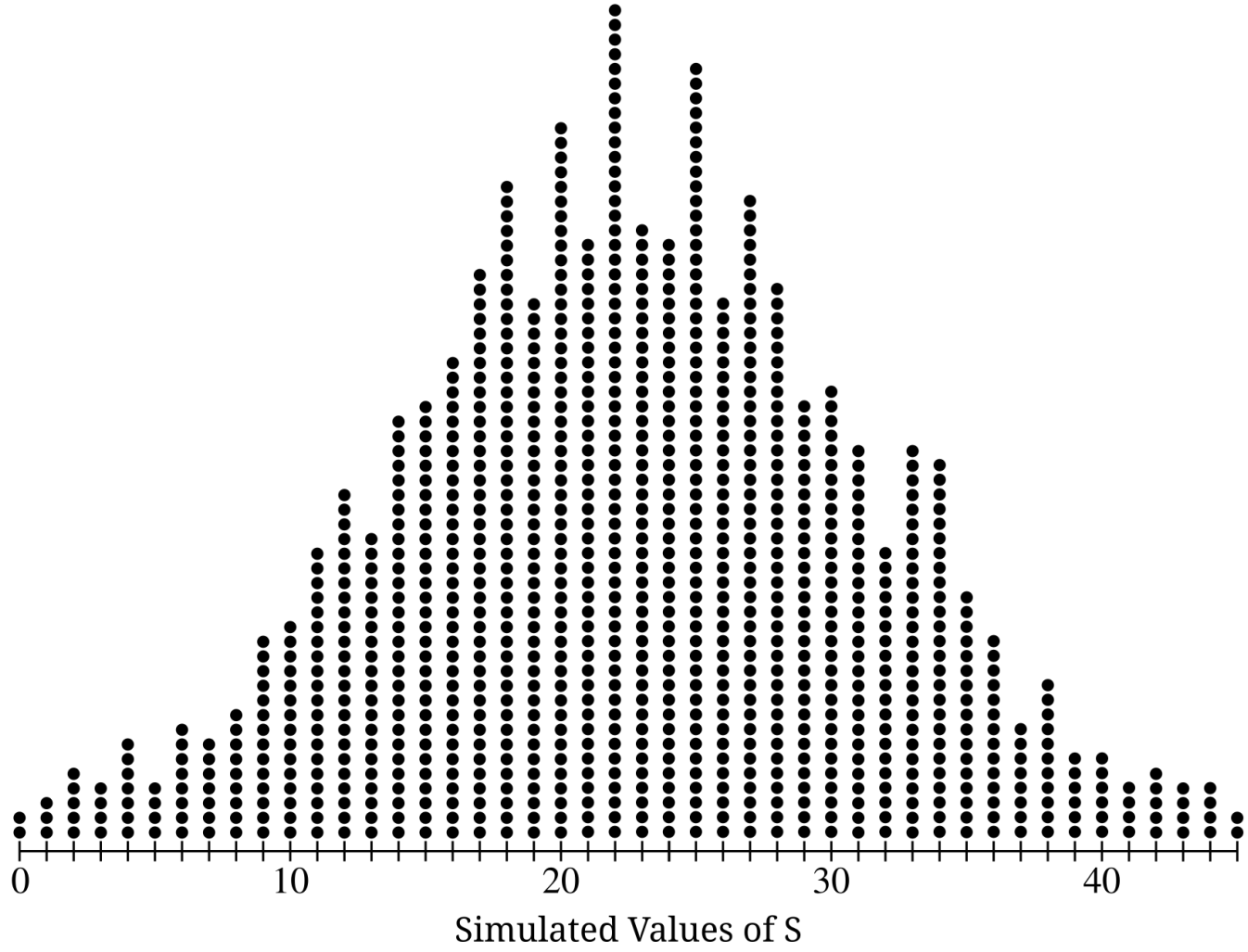
ii. Based on your answer in part (b-i), is it reasonable to believe that runners tend to have longer running times while not listening to music? Explain your answer without performing an inference procedure.

In part (b-ii), only the signs of the differences (positive or negative), not the magnitudes, are used. An alternative method that takes into account both the sign and the magnitude of the differences in running times is based on the sum, S, of ranks corresponding to positive differences. For the experiment described above, S is computed by first ordering the absolute values of the nine differences in times from smallest to largest. Next, ranks are computed for the absolute values of the differences with rank 1 corresponding to the smallest absolute value, rank 2 corresponding to the second smallest absolute value, up to rank 9 corresponding to the largest absolute value. Finally, the value of S is computed by summing only the values of the ranks corresponding to positive differences; the values of the ranks corresponding to negative differences are not included in the sum. The following table displays the differences and the ranks of the absolute values of the differences for the nine runners who participated in the experiment, where rank 1 indicates the smallest absolute value and rank 9 indicates the largest absolute value.

Differences	5	-11	-15	30	44	45	52	53	60
Ranks of absolute values of the differences	1	2	3	4	5	6	7	8	9

(c) Determine the value of S.

A simulation was carried out to investigate the sampling distribution of S. The value of S was calculated for each of 1,000 trials of the simulation, where each trial was simulated under the assumption that listening to music has no effect on running time. The results are shown in the dotplot below.



(d) Based on the simulation results and your response to part (c), is it reasonable to conclude that not listening to music tends to result in longer running times? Justify your conclusion.

STOP
END OF EXAM