AP® Statistics Exam

SECTION I: Multiple Choice

2021

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
40
Percent of Total Score
50%
Writing Instrument
Pencil required
Electronic Device
Graphing calculator expected

Instructions

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4PBP4-S

90

Formulas begin on page 3.
Questions begin on page 6.
Tables begin on page 42.

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\Sigma\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean u and standard deviation o, then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Formulas

(I) Descriptive Statistics

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If $\overline{x}$ is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ, then:

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic $\pm$ (critical value) $\bullet$ (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

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Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Statistics 2021Intl MCQ

1.

0	5
1	45668
2	3334
3	2449
4	2456
5	0 4 49
6	456
7	47

Key: $7|4 = 74\%$

In 2011, the Gallup polling organization selected random samples of residents from each of the 27 countries in the European Union and found the percentage of residents in each country who consider themselves to be thriving. The 27 percentages are shown in the stemplot.

What is the interquartile range for this data set?

(A) 27
(B) 31
(C) 39
(D) 41
(E) 72

2. The final scores for 309 participants in a national golf tournament are summarized in the following table.

N	Mean	SE Mean	St Dev	Min	Q1	Median	Q3	Max
309	149.35	0.37	6.42	131	145	150	153.5	168

Which of the following is closest to the z-score of the minimum golf score?

(A) -2.16
(B) -2.86
(C) -2.96
(D) -18.40
(E) -278.10

Statistics 2021Intl MCQ

3. A random sample of middle school students from District 1 and a random sample of middle school students from District 2 were selected with an even number of students in each sample. The number of minutes each student spent playing a certain mobile game application in a typical day was recorded. The data are summarized in the following table.

	Min	Q1	Med	Q3	Max
District 1	112	120	130	137	145
District 2	112	128	130	145	152

If no two students within each district spent the same amount of time playing the mobile game application, which statement must be true?

(A) The number of students in the sample from District 1 was less than the number of students in the sample from District 2.

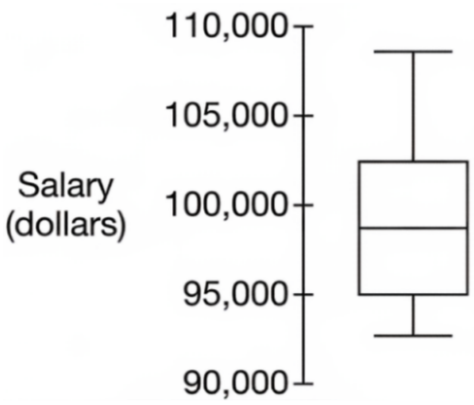
(B) The percentage of students in both samples who spent less than 130 minutes playing the game is equal.

(C) No students in either district spent 115 minutes playing the game.

(D) The range of the number of minutes spent playing the game was greater for the District 1 sample than for the District 2 sample.

(E) The interquartile range of the number of minutes spent playing the game was less for the District 1 sample than for the District 2 sample.

4. The boxplot shown summarizes the 100 highest salaries for municipal employees in a large city.

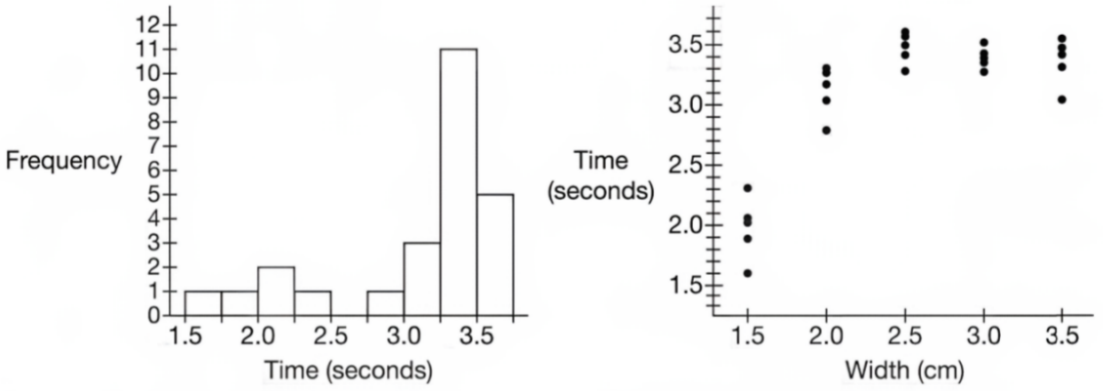


A new municipal employee was hired and was told that the salary would be within the middle 50 percent of the current 100 highest salaries.

Based on the boxplot, which of the following is closest to the width of the interval in which the new employee's salary will fall?

(A) $1,000
(B) $3,000
(C) $7,000
(D) $10,000
(E) $15,000

5. Students in a physics class designed an experiment to investigate the effect of blade width on flight time for paper helicopters. A histogram of the flight times, in seconds, and a scatterplot of flight time versus blade width, in centimeters (cm), are shown for 25 helicopters.



Which of the following best describes the clusters and the gap in the histogram?

(A) The left cluster displays flight times for blade widths from 1.5 cm to 2.5 cm, and the right cluster displays flight times for the remaining blade widths.

(B) The left cluster displays flight times for blade widths of 1.5 cm, and the right cluster displays flight times for the remaining blade widths.

(C) The left cluster displays minimum flight times for each of the blade widths, and the right cluster displays the remaining flight times.

(D) The clusters are not related to the blade widths because flight time is measured in seconds and blade widths are measured in centimeters.

(E) The clusters are not related to the blade widths because the relationship between blade width and flight time is not linear.

Statistics 2021Intl MCQ

6. A company conducted a charity drive and recorded the donations made by the employees of two departments, A and B. The back-to-back stemplot below shows the amounts given by the employees from department A and the amounts given by the employees from department B.

Department A		Department B
0 0 0 0	0	0 0 0 0 0 0 0 0
5 5 5 0 0 0	1	0 0 0 0 0 0 0 0 0 5 5 5 5 5
5 5 5 5 0 0	2	0 0 0 0 0 0 0 0 0 0 0 0 0 0 5 5 5 5 5 5 5
5 5 0	3	0 0 0 0 5 5 5 5 5 5 5
5 5 0 0 0	4	0 0 0
5 5 5 0 0 0 0	5	0 0 0 0
5 5 5	6	
5 5 5 5 5	7	5
5 5 5 5	8	
5 5 5 5 5 5 5 5 5 5 5 5 0 0 0 0 0 0 0	9	

Key: 7|5 represents $75

Based on the back-to-back stemplot, which of the following is correct?

(A) The variability of the donations for the employees from department A is less than the variability of the donations for the employees from department B.

(B) The median donation for the employees from department A is greater than the median donation for the employees from department B.

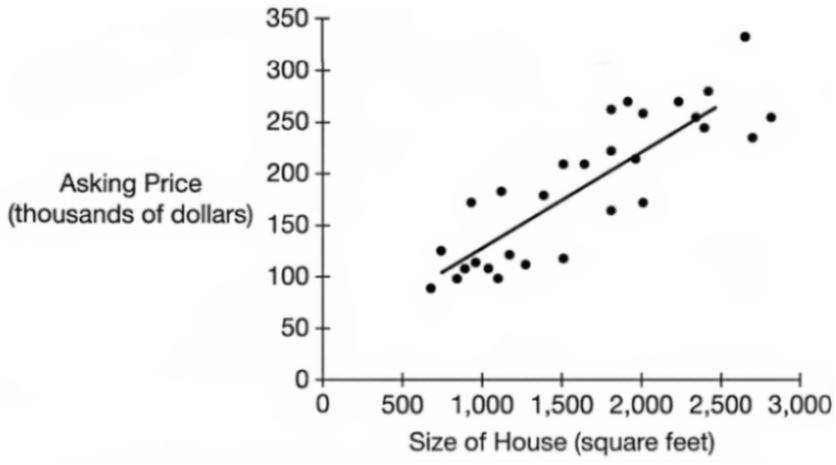
(C) The shape of the distribution of donations for each department is skewed left.

(D) The number of employees from department A who donated at most $35 is greater than the number of employees from department B who donated at most $35.

(E) The number of employees from department A who donated $50 is greater than the number of employees from department B who donated $50.

Statistics 2021Intl MCQ

7. A realtor collected data from a random sample of 29 houses for sale in a certain town. The scatterplot shows the size of each house, in square feet, and the asking price, in thousands of dollars. The least-squares regression line is also shown.



Based on the recorded data, the realtor created the following regression equation to predict asking price, in thousands of dollars, based on the size, in square feet, of the house.

$$\text{predicted asking price} = 35.050 + 0.096(\text{size of house})$$

Which of the following provides the best interpretation of the slope of the regression line?

(A) For each additional square foot, the predicted asking price of a house in this town increases by $\$96$.

(B) For each additional square foot, the predicted asking price of a house in this town increases by $\$35$.

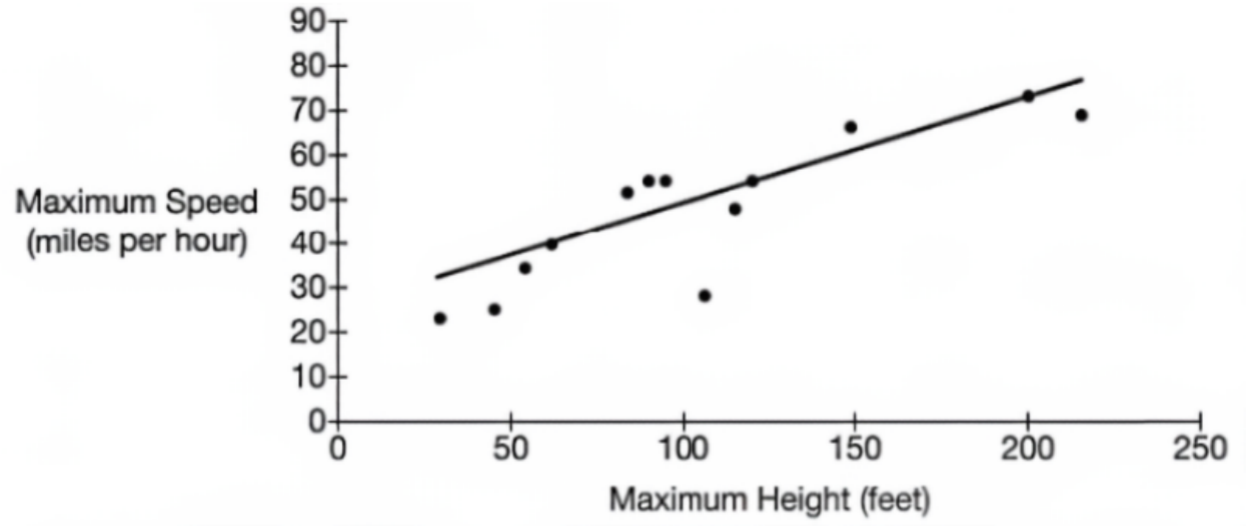
(C) For each additional square foot, the predicted asking price of a house in this town increases by $\$0.096$.

(D) For each additional $\$1{,}000$ of asking price, the predicted size of a house increases by 35 square feet.

(E) For each additional $\$1{,}000$ of asking price, the predicted size of a house increases by 0.096 square feet.

Statistics 2021Intl MCQ

8.



An engineering student collected data on the maximum height, in feet, and maximum speed, in miles per hour, of thirteen roller coasters. A scatterplot of the data and the least-squares regression line are shown. The maximum heights of five of the roller coasters are 60, 105, 150, 200, and 215 feet.

If the least-squares regression line is used to predict the maximum speed for the five roller coasters, for which maximum height, in feet, would the absolute value of the residual be largest?

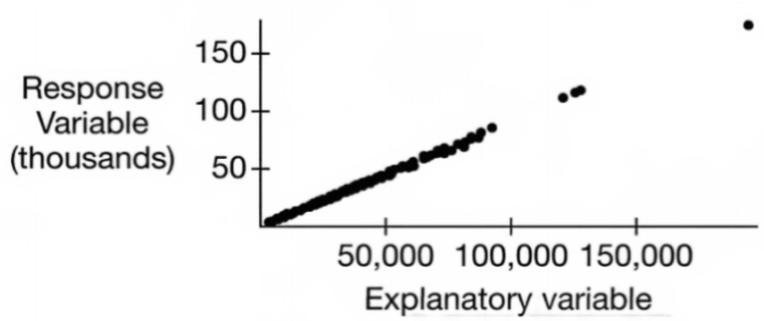
(A) 60
(B) 105
(C) 150
(D) 200
(E) 215

9. One condition for inference for slope of a regression line is that the standard deviation of the response variable does not vary with the explanatory variable.

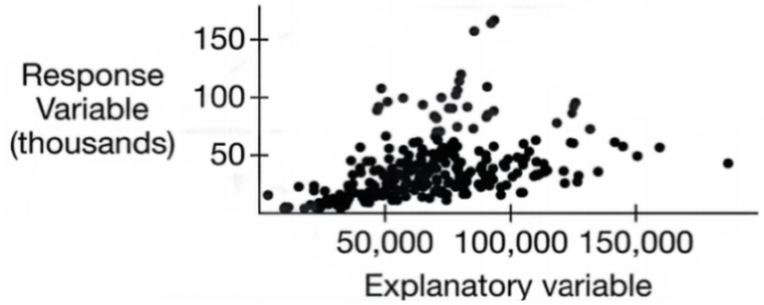
Which of the following scatterplots most likely exhibits a violation of that condition?

Statistics 2021Intl MCQ

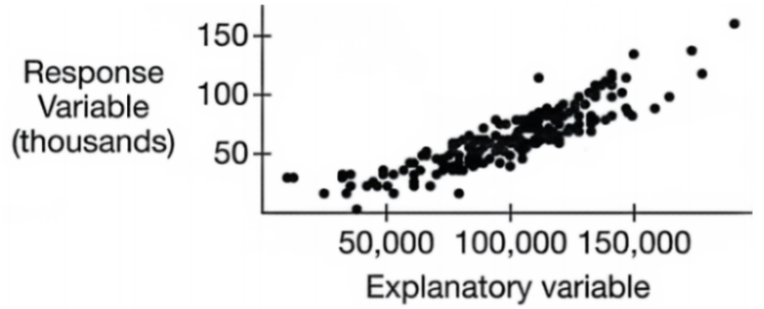
(A)



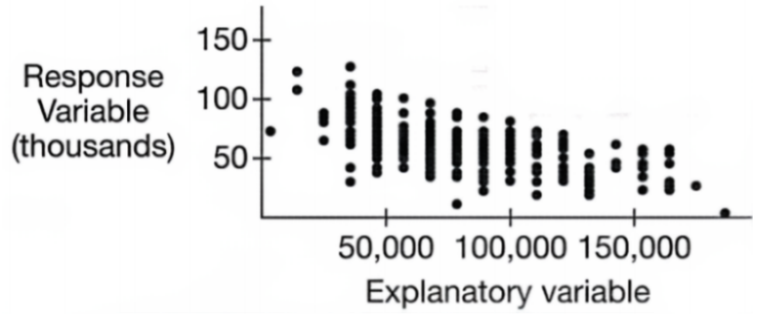
(B)



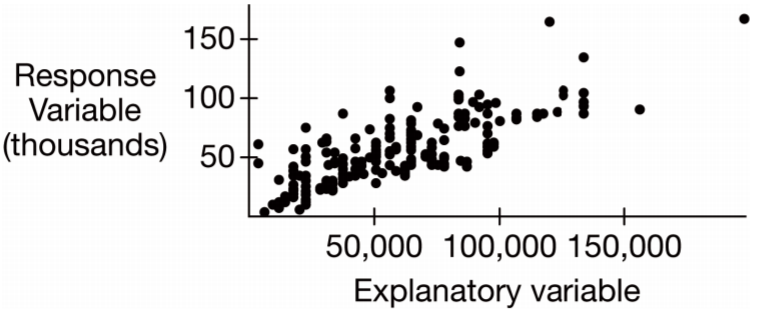
(C)



(D)

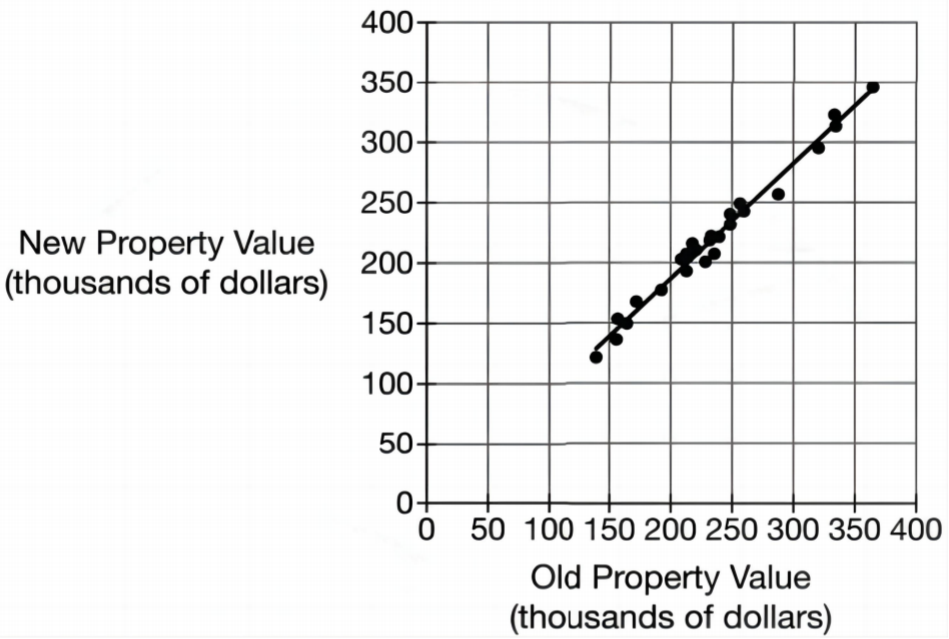


(E)



Statistics 2021Intl MCQ

10. A certain state law requires residential properties to receive an appraisal for tax purposes every six years. A random sample of 25 appraised properties was selected. The following scatterplot shows the value of the properties, in thousands of dollars, before and after the appraisal. Also shown is the least-squares regression line and corresponding computer output.



Term	Coef	SE Coef	T-Value	P-Value
Constant	-3.817	1.23	-0.60	0.5562
Old Property Value	0.95597	0.02634	36.30	< 0.0001
$S = 7.53867$	R-Sq $= 98.28\%$		R-Sq(adj) $= 98.21\%$	

Which of the following is not an appropriate description of the data in the sample?

(A) New property values tend to be greater than old property values.

(B) As old property values increase, new property values tend to increase.

(C) The relationship between old and new property value is strong and positive.

(D) For each additional $\$1{,}000$ in old property value, the new property value increases by $\$955.97$, on average.

(E) The observed new property values typically deviate from the predicted new property values by about $\$7{,}539$.

Statistics 2021Intl MCQ

11. A medical researcher conducted a study to investigate the effectiveness of two drugs, L and M, designed to reduce the number of seizures that people with epilepsy might have. Each person in a group of 120 adult volunteers who had epilepsy was randomly assigned to one of two groups. Those assigned to one group took drug L for one year, and those assigned to the other group took drug M for one year. Each participant recorded the number of seizures that occurred during the year.

To which group is it most reasonable to generalize the results of this study?

(A) All adults

(B) All adults with epilepsy

(C) All adults with epilepsy who take drugs to treat the disease

(D) All adults with epilepsy who are similar to those who volunteered for the study

(E) All adults who experience a reduction in the number of seizures

12. Researchers conducted a study to investigate whether watching an outdoor scene while exercising improves health more than exercise alone. Fourteen fifth-grade students participated in two trials of the study. For one trial, the students rode a stationary bicycle for 15 minutes while watching a video of someone cycling through a forest. For the other trial, the students rode a stationary bicycle for 15 minutes without watching a video. Each student completed both trials in a randomized order. The researchers recorded the students' heart rates after each trial.

Which of the following is the best description of the study?

(A) A randomized matched-pairs experiment

(B) A randomized placebo-controlled experiment

(C) A completely randomized experiment

(D) A sample survey

(E) An observational study

13. A new laser-guided surgical procedure for removing an inflamed appendix is believed to have a shorter recovery time for patients than an alternative surgical procedure that is not guided by laser has. In an experiment to evaluate recovery time, patients needing the surgery will be randomly assigned to one of two surgical procedures. A surgeon will perform the surgery for one group using lasers and for the other group without the use of lasers. Both types of surgeries are performed with similarly sized incisions and patients are unaware of which procedure is performed.

What type of blinding can be accomplished in the experiment?

(A) Both the patient and the surgeon can be blind to the treatment.

(B) Neither the patient nor the surgeon can be blind to the treatment.

(C) The patient can be blind to the treatment, but not the surgeon.

(D) The surgeon can be blind to the treatment, but not the patient.

(E) Blinding is not possible because the design has no control group.

14. A medical researcher is interested in assessing the effectiveness of a new medication for pain relief. Three hundred volunteers will be randomly assigned to groups. One group will receive the new medication at a low dose, one group will receive the new medication at a high dose, and one group will receive a placebo that looks like the new medication but does not contain any active ingredients. Each person will be asked to rate his or her level of pain at the beginning of the experiment and again two hours after taking the medication or placebo.

What is the number of treatments for this experiment?

(A) 2
(B) 3
(C) 6
(D) 100
(E) 300

15. The height requirement for a certain ride at an amusement park is 48 inches or taller. The distribution of height of six-year-old children is approximately normal with mean 45.2 inches and standard deviation 1.75 inches.

Which of the following is closest to the probability that a randomly selected six-year-old child will not meet the height requirement for the ride?

(A) 0.0401
(B) 0.0548
(C) 0.9452
(D) 0.9599
(E) 0.9974

16. A manufacturer of a certain item knows that 1 percent of the items produced on a certain assembly line are defective. Suppose that 100 items produced on that assembly line will be selected at random.

Which of the following expressions represents the probability that 3 of the 100 items selected will be defective?

(A) $\binom{100}{3}(0.01)^3(0.99)^{97}$
(B) $\binom{100}{97}(0.01)^{97}(0.99)^3$
(C) $\binom{97}{3}(0.01)^3(0.99)^{97}$
(D) $\binom{97}{3}(0.01)^{97}(0.99)^3$
(E) $\binom{100}{3}(0.01)^3(0.99)^3$

Statistics 2021Intl MCQ

17. The distribution of sale price for a certain model of car is approximately normal with mean $\$22{,}500$ and standard deviation $\$600$.

Based on the distribution, which statement must be true?

(A) Approximately 99% of the cars have a sale price less than $\$23{,}700$.

(B) Approximately 84% of the cars have a sale price greater than $\$21{,}900$.

(C) Approximately 95% of the cars have a sale price between $\$21{,}900$ and $\$23{,}100$.

(D) The interquartile range of the sale prices is approximately $\$600$.

(E) The minimum sale price is $\$20{,}700$.

18.

Number of cookies sold	50	100	200
Probability	0.60	0.35	0.05

The probability distribution of the number of cookies sold by a store each day is shown in the table. The store earns a profit of $\$0.50$ per cookie sold.

What is the store's expected daily profit for the sales of the cookies?

(A) $\$37.50$

(B) $\$75.00$

(C) $\$116.67$

(D) $\$150.00$

(E) $\$175.00$

19. An elevator has a posted weight limit of $4{,}200$ pounds and an occupancy limit of 21 people. If the weights of adult males are approximately normally distributed with $\mu = 190$ pounds and $\sigma = 32$ pounds, which of the following best approximates the probability that a random sample of 21 adult males will exceed the posted weight limit of the elevator?

(A) 0.001

(B) 0.076

(C) 0.155

(D) 0.378

(E) 0.500

Statistics 2021Intl MCQ

20. Clark drives through two traffic lights on his way to work each day. From experience, he estimates that the probability that the first light will be red when he reaches it is 0.3. He also estimates that the probability that the second light will be red when he reaches it is 0.45. The lights are far enough apart that they can be considered to operate independently of each other.

Based on Clark's estimates, what is the probability that at least one of the two lights will be red on a randomly selected drive to work?

(A) 0.135
(B) 0.165
(C) 0.385
(D) 0.615
(E) 0.750

21. The driving route between a student's home and the student's high school has two traffic lights. The distribution of total time spent driving has mean 5 minutes and standard deviation 1 minute. The distribution of time spent waiting at a light has mean 0.75 minute and standard deviation 0.25 minute. The total time spent driving and the time spent waiting at a light are independent. The total travel time from the student's home to the student's high school consists of the total time spent driving and the time spent waiting at two lights.

What are the mean and the standard deviation of the total travel time?

(A) The mean is 5.75 minutes and the standard deviation is about 1.03 minutes.
(B) The mean is 5.75 minutes and the standard deviation is about 1.06 minutes.
(C) The mean is 6.5 minutes and the standard deviation is about 1.06 minutes.
(D) The mean is 6.5 minutes and the standard deviation is about 1.12 minutes.
(E) The mean is 6.5 minutes and the standard deviation is about 1.5 minutes.

22. A six-week training program was developed to help student athletes improve performance on a certain task. To study the effectiveness of the program, researchers randomly selected 50 student athletes. The students were tested before participating in the program. Then, half the athletes were randomly assigned to participate in the program and the other half were allowed to train on their own. At the end of the six weeks, all athletes were tested again. The researchers reported that the athletes who participated in the program increased their scores, on average, by 200 points, whereas those who did not participate raised their scores, on average, by 80 points.

After reading the results of the study, Jasmine, a student athlete, decided to enroll in the program. At the end of six weeks, her score had increased by 80 points. Which of the following is an explanation for the difference between Jasmine's score and the scores that the study reported for student athletes who participated in the program?

(A) The results of the study were false; there must have been a bias in the sample that was not reported.
(B) Jasmine probably did not follow the program correctly; if she had, she would have seen a 200-point increase in her score.
(C) The increase of 200 points reported was an average; some athletes could have increased their scores by more than 200 points, and some by less than 200 points.
(D) The sample sizes used in the study were too small for any conclusions about the training to be valid.
(E) The study was not a well-designed experiment because causation cannot be attributed to the program.

23. Which of the following statements illustrates the Law of Large Numbers?

I. For n flips of a fair coin with one side heads, the proportion of flips that land heads up approaches 0.5 as n increases.

II. For n tosses of a fair six-sided die with one face labeled 1, the proportion of tosses that land with the face of 1 up approaches $\frac{1}{6}$ as n increases.

III. If each of 10 consecutive tosses of a fair coin resulted in a head, the probability that the next toss of the coin will result in a tail is greater than 0.5.

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

24. A 95 percent confidence interval for the mean weight, in thousands of pounds, of African bush elephants is determined to be $(12.2, 14.6)$.

Which of the following is a correct interpretation of the interval?

(A) We are 95% confident that 95% of African bush elephants weigh between 12.2 thousand pounds and 14.6 thousand pounds.

(B) The probability is 0.95 that the population mean weight of African bush elephants is between 12.2 thousand pounds and 14.6 thousand pounds.

(C) We are 95% confident that the population mean weight of African bush elephants is between 12.2 thousand pounds and 14.6 thousand pounds.

(D) We are 95% confident that the population mean weight of African bush elephants is 13.4 thousand pounds, which is the midpoint of the interval.

(E) We are 95% confident that the mean weight calculated from a different sample of African bush elephants would be between 12.2 thousand pounds and 14.6 thousand pounds.

25. Two brands of golf cart batteries, R and S, were tested to investigate whether there is a difference between the proportions of fully charged batteries that could power a golf cart for at least thirty miles. A random sample of 50 brand R batteries was selected and a random sample of 50 brand S batteries was selected. Batteries were randomly assigned to one of 100 different golf carts. Each cart was driven until the battery failed. Of the carts with brand R batteries, 40 traveled over thirty miles, and of the carts with brand S batteries, 35 traveled over thirty miles.

Which of the following is a 95 percent confidence interval for the difference in the proportions of fully charged batteries that can power a golf cart for at least thirty miles for the two brands?

Statistics 2021Intl MCQ

(A) $(0.8 - 0.7) \pm 1.645\sqrt{\frac{(0.8)(0.2)}{50} + \frac{(0.7)(0.3)}{50}}$

(B) $(0.8 - 0.7) \pm 1.645\sqrt{(0.75)(0.25)\left(\frac{1}{50} + \frac{1}{50}\right)}$

(C) $(0.8 - 0.7) \pm 1.645\sqrt{\frac{(0.8)(0.2)}{50} - \frac{(0.7)(0.3)}{50}}$

(D) $(0.8 - 0.7) \pm 1.960\sqrt{\frac{(0.8)(0.2)}{50} + \frac{(0.7)(0.3)}{50}}$

(E) $(0.8 - 0.7) \pm 1.960\sqrt{(0.75)(0.25)\left(\frac{1}{50} + \frac{1}{50}\right)}$

26. An agriculturalist wants to estimate the average time from germination to ripe fruit for a new type of seedless strawberry. From a random sample of 25 seedless strawberries of the new type, the mean time from germination to ripe fruit is 42.1 days with standard deviation 3.6 days.

Assume all conditions for inference were met. Which of the following is a 90 percent confidence interval for the mean time from germination to ripe fruit for the new type of seedless strawberry?

(A) 42 ± 6.16

(B) 42.1 ± 1.49

(C) 42.1 ± 1.23

(D) 42.1 ± 1.18

(E) 42.1 ± 0.25

27. Researchers selected a random sample of 100 students from a certain college to investigate whether students from the college who are sleep deprived will have lower reading-comprehension scores, on average, than students from the college who are not sleep deprived. Half the students from the sample were randomly assigned to a treatment group that received sleep restrictions, and the other half were assigned to a control group that received no sleep restrictions. The students were given a reading-comprehension test before the study began and at the end of two days. The mean score of the test for the treatment group was similar to that of the control group before the study. A 95 percent confidence interval constructed from the difference in mean scores (treatment minus control) at the end of two days was $(-7.3, -1.5)$.

Based on the confidence interval, which of the following is an appropriate conclusion?

(A) The data provide no evidence that the population of sleep-deprived students from the college has a lower mean reading-comprehension score than the population of students from the college who are not sleep deprived.

(B) The data provide convincing statistical evidence that the population of sleep-deprived students from the college has a lower mean reading-comprehension score than the population of students from the college who are not sleep deprived.

(C) No conclusion can be made because a confidence interval cannot be used to assess statistical evidence.

(D) The probability that a student from the college who is sleep deprived will have a lower reading-comprehension score than a student from the college who is not sleep deprived is 0.95.

(E) More than half the students from the college do worse on a reading-comprehension test after being sleep deprived.

Statistics 2021Intl MCQ

28. An engineer wanted to investigate the relationship between total stopping distance required (reaction distance and actual breaking distance) and speed of a car. The engineer completed an experiment measuring the total stopping distance of the car at 5 different speeds, and the resulting regression output is shown below.

Predictor	Coef	SECoef	T	P
Constant	-284.29	63.032	-4.51	0.0204
Speed	11.905	1.220	9.76	0.0023
$S = 35.56$	R-sq $= 96.9\%$		R-sq (adj) $= 95.9\%$	

Assume the conditions for inference are met. Which of the following represents a 95 percent confidence interval for the slope of the regression line?

(A) $11.905 \pm 3.182(1.220)$

(B) $11.905 \pm 3.182\left(\frac{1.220}{\sqrt{5}}\right)$

(C) $11.905 \pm 3.182\left(\frac{1.220}{\sqrt{4}}\right)$

(D) $11.905 \pm 2.776(1.220)$

(E) $11.905 \pm 2.776\left(\frac{1.220}{\sqrt{5}}\right)$

29. To estimate the proportion of Europeans who walk for exercise at least 10 minutes a week, data were collected from a random sample of $27{,}919$ people from Europe. The 95 percent confidence interval created from the data was $(0.856, 0.864)$.

Which of the following is an appropriate interpretation of the confidence interval?

(A) The probability that a randomly selected person from Europe walks for exercise at least 10 minutes a week is 0.95.

(B) The proportion of Europeans who walk between 0.856 and 0.864 minute a week for exercise is 0.95.

(C) There is a 95% chance that the sample proportion of Europeans who walk for exercise at least 10 minutes a week is between 0.856 and 0.864.

(D) Between 85.6% and 86.4% of all Europeans walk for exercise at least 10 minutes a week.

(E) We can be 95% confident that the proportion of all Europeans who walk for exercise at least 10 minutes a week is between 0.856 and 0.864.

Statistics 2021Intl MCQ

30. A study was conducted to investigate whether there is a difference in the mean life span of a new brand of battery compared with an older brand of battery. Independent random samples of 9 new-brand batteries and 8 older-brand batteries were selected, and the life span (in hours) of each battery was recorded. Summary statistics obtained from the data are shown in the table below. Assume that the conditions for inference are met.

	New Brand	Older Brand
Mean	426.8	355.2
Standard Deviation	49.5	96.7

Which of the following best approximates the p-value for a test of $\mathrm{H}_0 : \mu_{\text{new}} - \mu_{\text{older}} = 0$ versus $\mathrm{H}_\mathrm{a} : \mu_{\text{new}} - \mu_{\text{older}} \neq 0$, where μ_{new} is the population mean life span of the new brand of battery and μ_{older} is the population mean life span of the older brand of battery?

(A) $P\left(t > \frac{426.8-355.2}{\sqrt{\frac{49.5}{9}+\frac{96.7}{8}}}\right)$

(B) $P\left(t > \frac{426.8-355.2}{\sqrt{\frac{49.5^2}{9}-\frac{96.7^2}{8}}}\right)$

(C) $P\left(t > \frac{426.8-355.2}{\sqrt{\frac{49.5^2}{9}+\frac{96.7^2}{8}}}\right)$

(D) $2 \cdot P\left(t > \frac{426.8-355.2}{\sqrt{\frac{49.5}{9}+\frac{96.7}{8}}}\right)$

(E) $2 \cdot P\left(t > \frac{426.8-355.2}{\sqrt{\frac{49.5^2}{9}+\frac{96.7^2}{8}}}\right)$

Statistics 2021Intl MCQ

31. The campaign manager for a candidate for governor wants to estimate the difference between the percentage of voters who viewed an advertisement and favor the candidate and the percentage of voters who had not viewed the advertisement and favor the candidate. Two independent random samples of 500 voters were selected. One random sample consisted of voters who viewed the advertisement and the other random sample consisted of voters who had not viewed the advertisement. The table shows the frequencies of responses for those surveyed.

	Have Not Seen Advertisement	Have Seen Advertisement
In favor	224	283
Not in favor	276	217

Which of the following is the most appropriate method for the campaign manager to use to estimate the difference between the percentages of voters who favor the candidate?

(A) A one-sample z-interval for estimating a population proportion

(B) A t-interval for estimating the slope of the least-squares regression line

(C) A matched-pairs t-interval for estimating a mean difference

(D) A two-sample z-interval for estimating a difference between sample proportions

(E) A two-sample z-interval for estimating a difference between population proportions

32. A national newspaper reported that 40 percent of high school students engage in community service activities. A school administrator wondered whether the proportion of high school students at the administrator's school who engage in community service activities is less than 40 percent. The administrator selected a random sample of 100 students, and 36 responded that they do engage in community service activities. The conditions for inference were met, and an appropriate hypothesis test resulted in a p-value of 0.207.

Which of the following is a correct interpretation of this p-value?

(A) The probability that a randomly selected student at this high school engages in community service activities is 0.207.

(B) The probability is 0.40 that less than 20.7 percent of students at this high school engage in community service activities.

(C) The probability is 0.207 that less than 40 percent of students at this high school engage in community service activities.

(D) If the proportion of all students at the school who engage in community service activities is 0.4, the probability of observing 36 or fewer in a random sample of 100 students who engage in community service activities is 0.207.

(E) If the proportion of all students at the school who engage in community service activities is 0.36, the probability of obtaining 40 or fewer in a random sample of 100 who engage in community service activities is 0.207.

Statistics 2021Intl MCQ

33. The editor of a local newspaper wants to know whether readers would be willing to pay an additional $\$1.00$ per month for their monthly newspaper subscription in exchange for an extra weekly section featuring stories about local businesses. The editor plans to survey a random sample of readers, and if the editor finds convincing statistical evidence that more than 75 percent of readers support the change, the editor will implement the plan of starting the extra weekly section.

In the context of this situation, which of the following describes a consequence of a Type I error?

(A) The editor decides to implement the plan of starting the extra weekly section when the actual percentage of readers who support the change is greater than 75%.

(B) The editor decides to implement the plan of starting the extra weekly section when the actual percentage of readers who support the change is not greater than 75%.

(C) The editor decides to implement the plan of starting the extra weekly section when the actual percentage of readers who support the change is less than 50%.

(D) The editor decides not to implement the plan of starting the extra weekly section when the actual percentage of readers who support the change is greater than 75%.

(E) The editor decides not to implement the plan of starting the extra weekly section when the actual percentage of readers who support the change is not greater than 75%.

34. Which statement about the power of a significance test is true?

(A) A test performed at a significance level of $\alpha = 0.01$ has a higher power than the same test performed at a $\alpha = 0.05$.

(B) If the probability of Type I error is decreased, the power of the test is increased.

(C) The power of a test is equal to the probability of making a Type II error.

(D) The power of a test is independent of the sample size.

(E) The power of a test at a particular parameter value increases as the distance from the hypothesized value to the parameter value increases.

35. A researcher in sports medicine is interested in comparing the amount of daily screen time between high school students who are athletes and high school students who are not athletes. A random sample of 100 high school students who are athletes and a random sample of 100 high school students who are not athletes will be selected. Each student will be asked "Do you typically have more than 3 hours of screen time each day?"

Which of the following is the most appropriate test for the researcher to use to investigate whether there is a difference in the amounts of daily screen time?

(A) A two-sample z-test for a difference between population proportions

(B) A two-sample t-test for a difference between population proportions

(C) A two-sample z-test for a difference between population means

(D) A two-sample t-test for a difference between population means

(E) A chi-square goodness-of-fit test

Statistics 2021Intl MCQ

36. Researchers studying the relationship between sleep and memory obtained a sample of 28 adults to participate in an experiment. Half of the participants were randomly assigned to a full night's sleep, and the other half were wakened every two hours throughout the night. The following day, the participants were given a list of words to memorize and recall after one hour. Researchers recorded the number of words recalled by each participant. The p-value of an appropriate statistical test was 0.29.

Which of the following is a reasonable conclusion?

(A) The data provide sufficient evidence that memory is related to sleep.

(B) The data provide sufficient evidence that memory is not related to sleep.

(C) A cause-and-effect relationship between memory and sleep cannot be established. However, the data provide sufficient evidence that there is an association between memory and sleep.

(D) The data do not provide convincing evidence that memory is related to sleep.

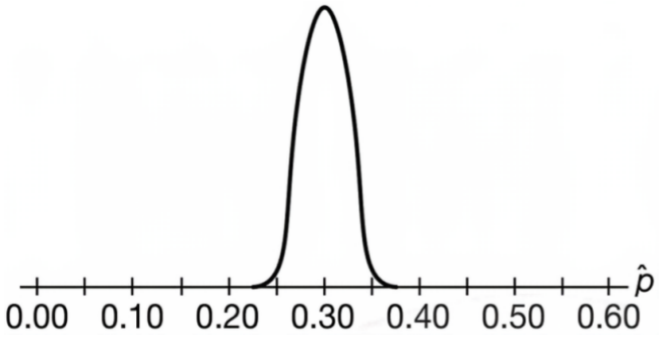
(E) The conditions for inference must not have been satisfied.

37. A recent report stated that 30 percent of smartphone users exceed their data limit in a given month. A consumer advocacy group questions the validity of the reported percent. The group obtained a random sample of 100 smartphone users and found that 40 of them exceeded their data limit in a certain month.

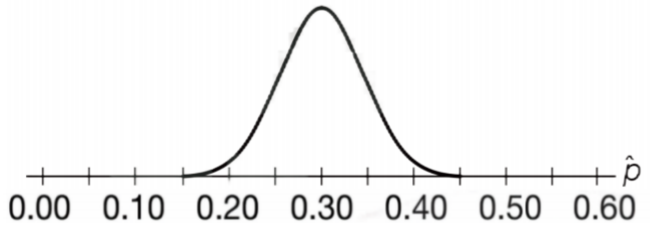
Of the following sampling distributions, which one should be used to test the hypothesis that the proportion is different than 30 percent?

Statistics 2021Intl MCQ

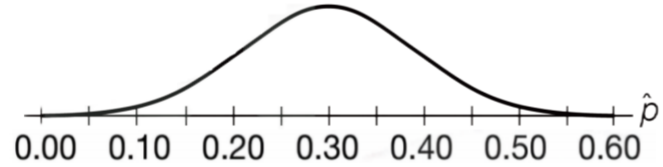
(A)



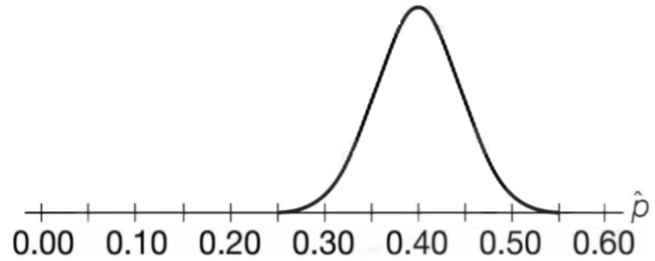
(B)



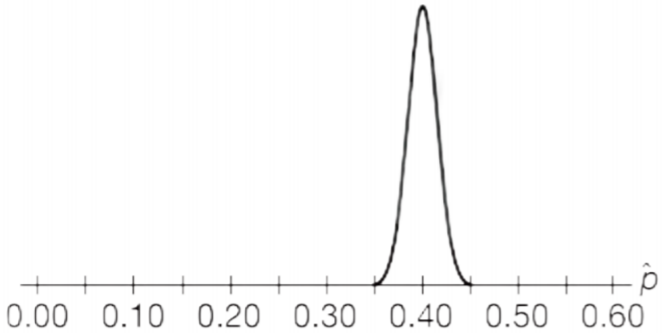
(C)



(D)



(E)



38. A college professor administered a survey to students from three different majors: engineering, nursing, and communications. The students were asked to identify how they prefer to be contacted: e-mail, telephone, or text message. The professor then used the survey data to conduct a chi-square test.

Which of the following best describes the null hypothesis for the test?

(A) There is an association between major and preferred method of communication.

(B) There is a positive association between major and preferred method of communication.

(C) There is no association between major and preferred method of communication.

(D) One-third of all students at the college are engineering majors, one-third are nursing majors, and one-third are communication majors.

(E) One-third of all students at the college prefer to be contacted by e-mail, one-third prefer to be contacted by telephone, and one-third prefer to be contacted via text message.

39. A random sample of students was selected at each of two high schools to investigate whether the proportions of students who drive themselves to school are different for the two schools. Of the 100 students sampled at one high school, 25 reported driving themselves to school. Of the 200 students sampled at the other high school, 62 reported driving themselves to school.

Which of the following is equivalent to the p-value for a test to determine whether there is convincing statistical evidence of a difference between the proportions of students who drive themselves to school for the two schools?

(A) $P\left(z > \frac{0.25-0.31}{\sqrt{\frac{(0.25)(0.75)}{100}+\frac{(0.31)(0.69)}{200}}}\right)$

(B) $P\left(z < \frac{0.25-0.31}{\sqrt{\frac{(0.25)(0.75)}{100}+\frac{(0.31)(0.69)}{200}}}\right)$

(C) $2 \cdot P\left(z < \frac{0.25-0.31}{\sqrt{\frac{(0.25)(0.75)}{100}+\frac{(0.31)(0.69)}{200}}}\right)$

(D) $P\left(z < \frac{0.25-0.31}{\sqrt{(0.29)(0.71)\left(\frac{1}{100}+\frac{1}{200}\right)}}\right)$

(E) $2 \cdot P\left(z < \frac{0.25-0.31}{\sqrt{(0.29)(0.71)\left(\frac{1}{100}+\frac{1}{200}\right)}}\right)$

40. A research agency surveyed 2,209 residents who live in one of three districts in a county: urban, suburban, or rural. The residents were asked to choose one of four categories that aligned with their opinion on how wireless infrastructure in the county should be paid for: wage tax, sales tax, property tax, or user fee.

When investigating whether opinion about how the wireless infrastructure should be paid for is independent of the district in which residents live, which of the following is the correct degrees of freedom for the appropriate chi-square test?

(A) 6

(B) 7

(C) 9

(D) 12

(E) 16

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

Table entry for z is the probability lying below z.

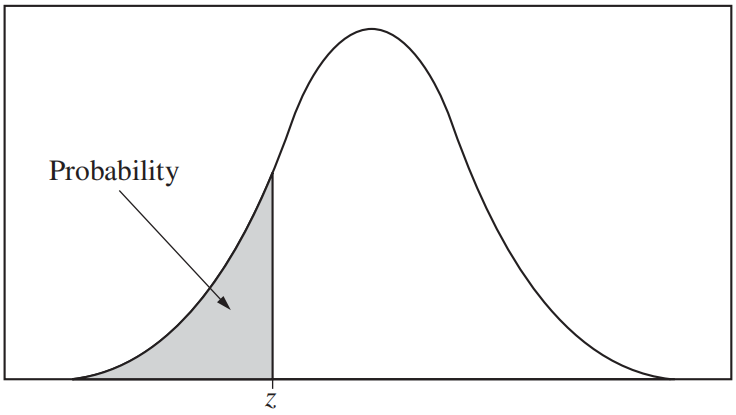


Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

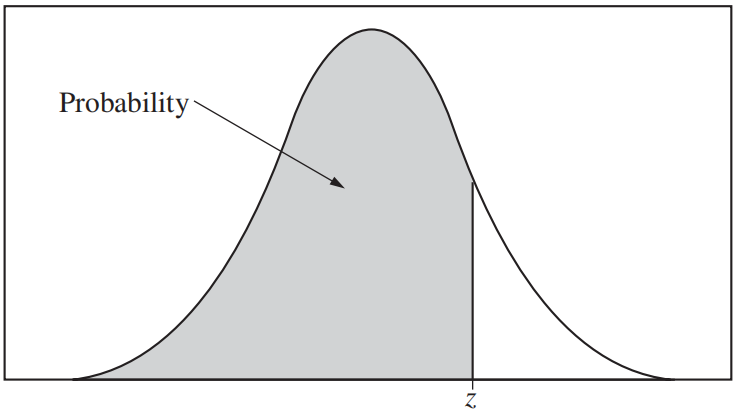


Table A (***Continued***)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

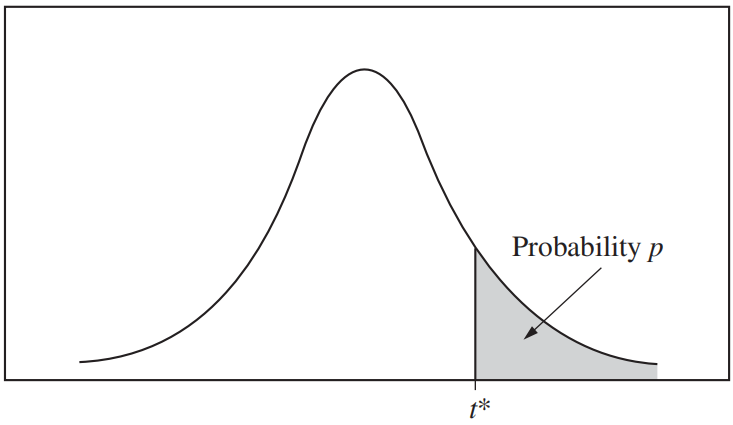


Table B t **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

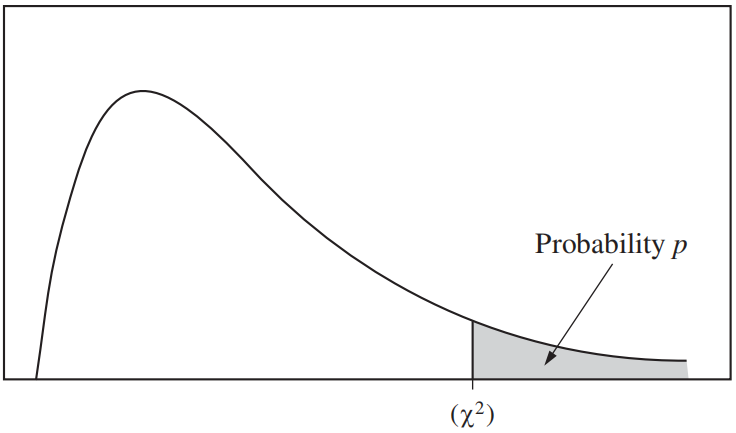


Table C χ^2 critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

AP® Statistics Exam

SECTION II: Free Response

2021

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Electronic Device
Graphing calculator expected

Part A

Number of Questions
5
Suggested Time
1 hour and 5 minutes
Percent of Section II Score
75%

Part B

Number of Questions
1
Suggested Time
25 minutes
Percent of Section II Score
25%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐

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Instructions

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I
Form Code 4PBP4-S

90

Formulas begin on page 3.
Questions begin on page 6.
Tables begin on page 22.

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\Sigma\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean u and standard deviation o, then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic $\pm$ (critical value) $\bullet$ (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

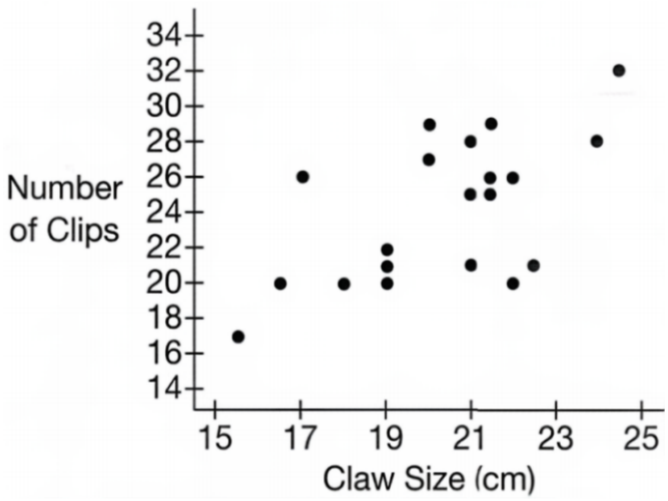
Statistic	Standard Deviation
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

Statistics 2021Intl MCQ

1. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A robotics company is developing a robotic claw that can grab paper clips. The company is investigating the number of clips that can be grabbed by the claw at one time based on the size of the claw. The following scatterplot shows the number of clips grabbed by 20 claws, with claw size measured as the span of the claw in centimeters (cm).



Two models have been proposed to predict the number of clips from claw size. Model I is given as the following.

$$\text{predicted number of clips} = 3.179 + 1.032(\text{claw size})$$

(a) Interpret the slope of Model I in context.

(b) Explain why the intercept of Model I is not meaningful in context.

(c) Model II has an intercept of 0 and is given as the following.

$$\text{predicted number of clips} = 1.190(\text{claw size})$$

(i) Consider the claw of size 18cm that grabbed 20 clips. Calculate the residual for this observation using both models. Show your work.

Model I residual:

Model II residual:

(ii) Which of the two models provides a better prediction for a claw of size 18cm that grabbed 20 clips? Justify your answer.

Statistics 2021Intl MCQ

2. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Researchers at a large university with a population of 20,000 students are investigating student opinions about the university fitness center. A list of the names of students and how often they use the fitness center is available.

Two methods of stratification are being considered for selecting a sample of 120 students.

Method I - Stratify by how often students use the fitness center (no usage, infrequent usage, and frequent usage).

Method II - Stratify by first letter of students' last names (A to F, G to M, N to S, and T to Z).

(a) Which of the two methods for creating the strata, method I or method II, should the researchers use for their investigation? Explain your answer.

(b) Describe a procedure that could be used to select a stratified random sample of 120 students at the university for the method selected in part (a). The procedure should result in a sample that consists of an equal number of students from each stratum.

Statistics 2021Intl MCQ

3. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A farm produces olive oil that is put into bottles that are sealed with a cork. The diameter of the bottle opening is 1.03 inches. To provide an acceptable fit, the cork diameter should be from 0.99 inch to 1.03 inches. The corks are purchased in bulk from a supplier, and the distribution of cork diameters is approximately normal with mean 1.00 inch and standard deviation 0.05 inch.

(a) If one cork is selected at random, what is the probability that the cork selected will provide an acceptable fit for the bottle? Show your work.

A farm manager decides to purchase the corks from a new supplier. Seventy percent of corks produced by the new supplier have a diameter with an acceptable fit.

(b) The farm manager is investigating how many corks in a box of 20 randomly selected corks will provide an acceptable fit. Consider the probability that at least 19 of the corks will provide an acceptable fit for the bottle.

(i) Define the random variable of interest and state how the random variable is distributed.

(ii) What is the probability that at least 19 of the corks will provide an acceptable fit for the bottle? Show your work.

(c) Suppose corks are inspected one at a time until an acceptable fit is found. Let the random variable W represent the number of corks inspected. What is the probability that W is less than or equal to 2 ? Show your work.

Statistics 2021Intl MCQ

4. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Disc golf is a sport in which players throw discs at a target. Atticus is investigating whether the weight of the disc used to play disc golf affects the accuracy of the throw. He obtained 15 standard discs weighing 175 grams each and 15 lightweight discs weighing 150 grams each. A player who is good at the game stood 40 yards from a target and threw each disc in random order. Atticus assessed the accuracy of each throw by measuring the distance, in feet, from where the disc landed to the target. The results and summary statistics are shown in the following table.

Accuracy at 175 Grams		
3.5	3.8	3.8
3.9	4.0	4.0
4.1	4.2	4.2
4.3	4.4	4.4
4.5	4.7	4.8

Accuracy at 150 Grams		
3.3	3.5	3.5
3.8	3.9	4.0
4.1	4.2	4.2
4.2	4.3	4.4
4.5	4.6	4.7

Accuracy at 175 Grams **Summary Statistics**	**Accuracy at 150Grams** **Summary Statistics**
$n = 15$	$n = 15$
$\bar{x} = 4.173$ feet	$\bar{x} = 4.080$ feet
$s = 0.356$ feet	$s = 0.416$ feet

Do the data provide convincing statistical evidence, at the level of $\alpha = 0.05$, that players tend to have more accuracy with one disc weight than with the other? Complete the appropriate inference procedure to support your answer.

Statistics 2021Intl MCQ

5. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

The instructor of a training course for a certain coding language hopes that a student's score on a pretest will predict the student's score on the certification final taken at the end of the course. The final scores indicate proficiency with the coding language. A random sample of 12 students who completed the pretest and the training course was selected, and the final scores of the students were recorded. The following tables show summary statistics and computer output for a regression analysis of the sample.

	Mean	Standard Deviation
Pretest	54.833	10.408
Final	78.867	6.946

Predictor	Coef	SE	T	P
Constant	49.2804	6.9199	7.12	0.0000
Pretest	0.5396	0.1242	4.35	0.0015
$S = 4.2682$		R-sq $= 65.38\%$		R-sq(adj) $= 61.92\%$

(a) Identify the equation of the least-squares regression line for predicting final score from pretest score. Identify the variables that appear in the equation in context.

(b) Consider the information presented in the tables and your answer from part (a).

(i) Determine the value of the predicted final score for students with a pretest score of 65.

(ii) Interpret $S = 4.2682$ in the context of the problem.

For any particular pretest score, two point estimates can be calculated from the given simple linear regression analysis.

Point Estimate I: An estimate of the mean final score among all students with a particular pretest score who complete the course.
Point Estimate II: A prediction for the final score of one student with a particular pretest score who completes the course.

The values of both point estimates are equal to the predicted value of the least-squares regression equation for the particular pretest score, such as the estimate from part (b-i) for a pretest score of 65. Although the values of the two point estimates are the same, the sampling variability is not the same.

(c) Which of the two point estimates has the larger sampling variability? Explain your reasoning.

(d) The value calculated in part (b-i) is an estimate of the mean final score for all students with a pretest score of 65 who completed the course. If a list of the pretest scores and final scores for all students who completed the course is obtained, explain how this information could be used to investigate the sampling distribution of the estimated mean final scores for all students with a pretest score of 65.

6. Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

A study was conducted with 50 volunteers to investigate whether yawning is contagious. Each volunteer was randomly assigned to one of two groups: Group A or Group B. Each person in Group A was shown a video in which someone yawned, and each person in Group B was shown a video in which no one yawned. Of the 34 people assigned to Group A, 11 yawned, and of the 16 people assigned to Group B, 3 yawned.

(a) Is this study an observational study or an experiment? Explain your choice in context.

Let p_A represent the proportion of all people who would yawn when shown the video in which someone yawned, and let p_B represent the proportion of all people who would yawn when shown the video in which no one yawned.

(b) Explain why it is inappropriate to use a two-sample z-test for proportions to test the following hypotheses.

$H_0 : p_A = p_B$

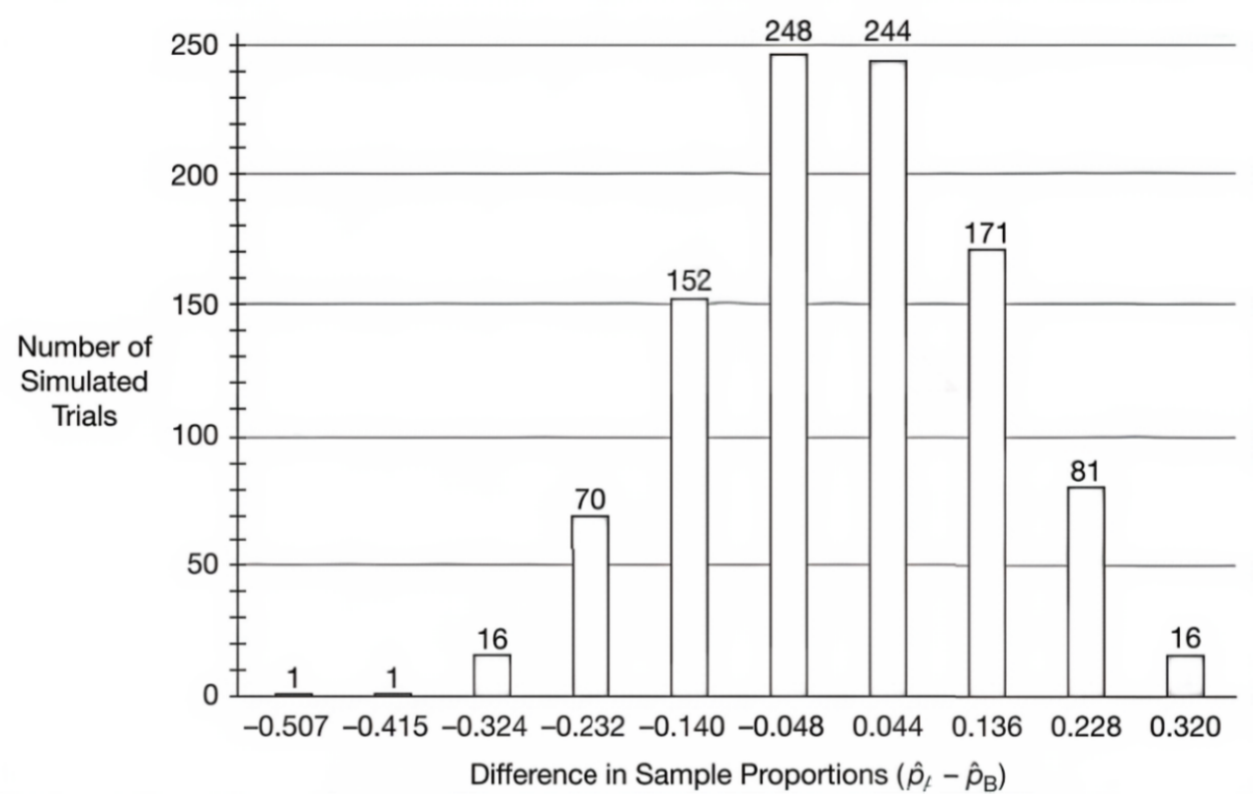
$H_a : p_A > p_B$

(c) Based on the study data, determine the point estimate for $p_A - p_B$. Show your work.

(d) A simulation to approximate the sampling distribution of the difference in sample proportions $\hat{p}_A - \hat{p}_B$ was conducted under the assumption that yawning is not contagious.

The graph shows the results of 1,000 trials of such a simulation.

Statistics 2021Intl MCQ



(i) Based on your answer in part (c) and the simulated results, what is the approximate p-value for the test stated in part (b)? Show your work.

(ii) Based on your answer to part (d-i) and using a significance level of 0.05, what conclusion is appropriate about whether yawning is contagious? Justify your answer.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Table entry for z is the probability lying below z.

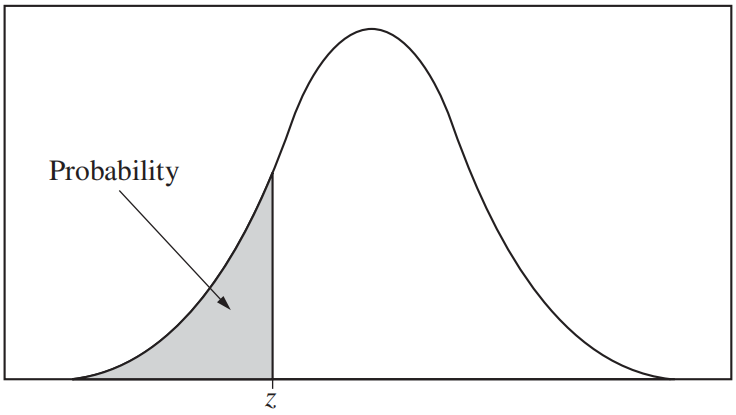


Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

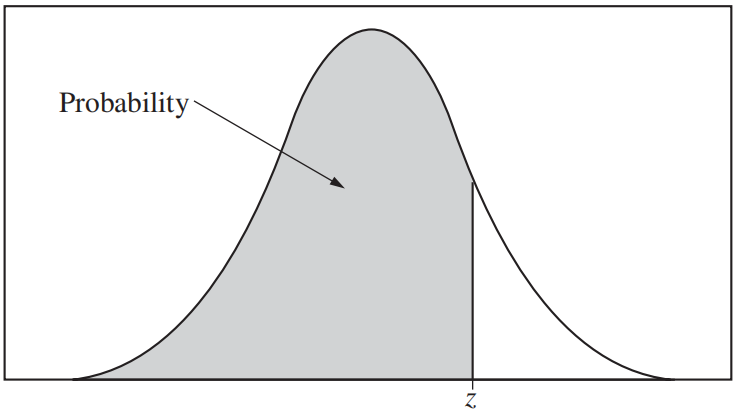


Table A (***Continued***)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

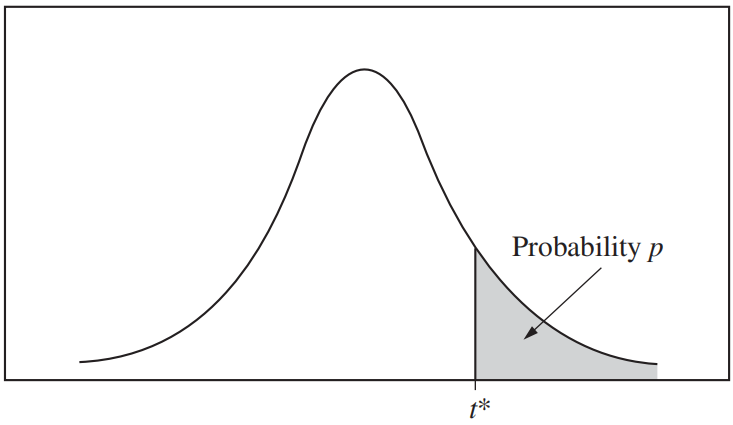


Table B t **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

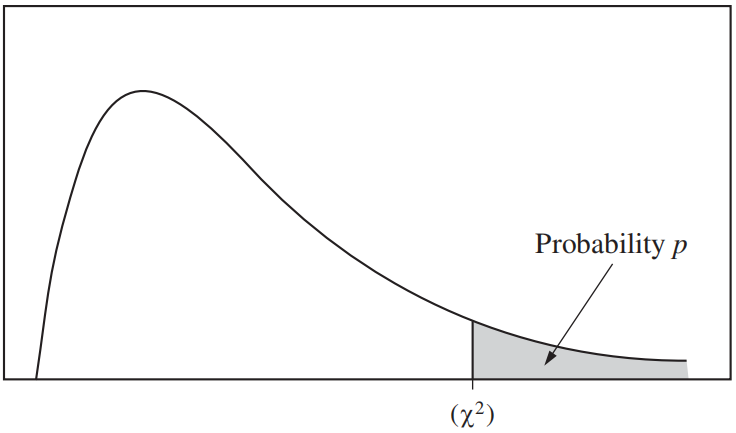


Table C χ^2 critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2