From the 2019 Administration

AP® CollegeBoard

AP® Statistics

Practice Exam

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AP® Statistics Exam

SECTION I: Multiple Choice

2019

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
40
Percent of Total Score
50%
Writing Instrument
Pencil required
Electronic Device
Graphing calculator expected

Instructions

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4PBP4-S

90

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\Sigma\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_X = \sum x_i p_i$$

$$\mathrm{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_X = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If i is the mean of a random sample of size n from an infinite population with mean u and standard deviation o, then:

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic $\pm$ (critical value) $\bullet$ (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

STATISTICS

SECTION I

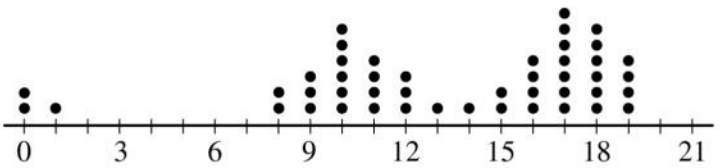
Time—1 hour and 30 minutes

Number of questions—40

Percent of total score—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. A group of students played a game in which they earned points for answering questions correctly. The following dotplot shows the total number of points earned by each student.



Which of the following is the best description of the distribution of points earned?

(A) Approximately normal

(B) Bimodal without a gap

(C) Bimodal with a gap

(D) Skewed to the right without a gap

(E) Skewed to the right with a gap

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2. The table shows the responses from 103 people when asked if they support a proposal to expand the public library.

	Under the Age of 55	Age 55 or Older	Total
Yes	17	8	25
No	42	36	78
Total	59	44	103

One person from those who responded will be selected at random. Which of the following is closest to the probability that the person selected will be someone who responded no, given that the person selected is age 55 or older?

(A) 0.350

(B) 0.427

(C) 0.462

(D) 0.757

(E) 0.818

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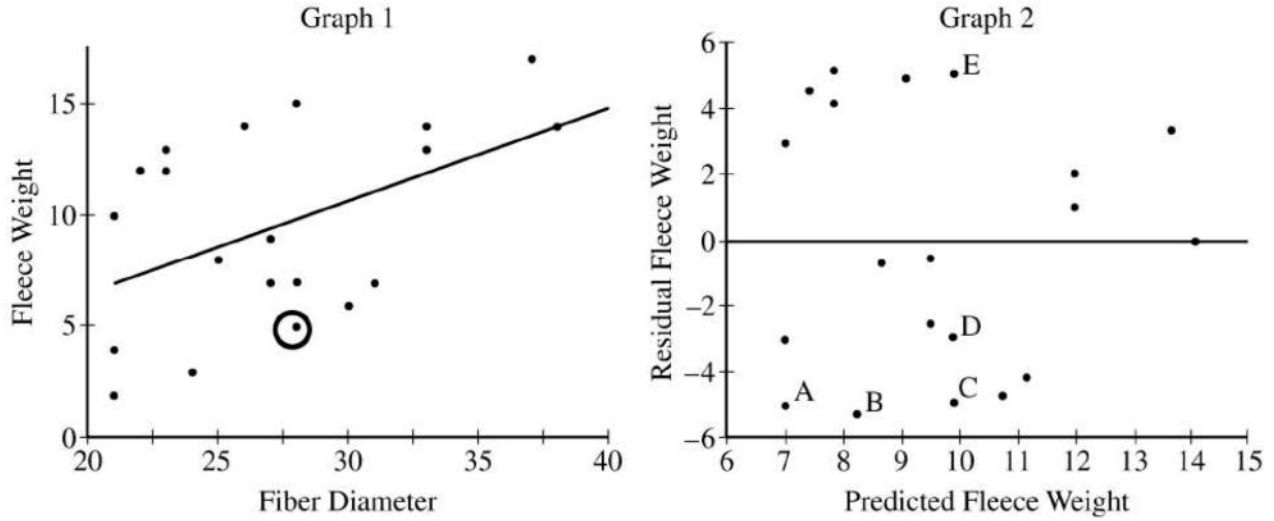
3. Data were collected on the fiber diameter and the fleece weight of wool taken from a sample of 20 sheep. The data are shown in the following graphs. Graph 1 is a scatterplot of fleece weight versus fiber diameter with the respective least-squares regression line shown. Graph 2 is the associated plot of the residuals versus the predicted values.



One point is circled on graph 1. Five points labeled A, B, C, D, and E are identified on graph 2. Which point on graph 2 represents the residual for the circled point on graph 1 ?

(A) A

(B) B

(C) C

(D) D

(E) E

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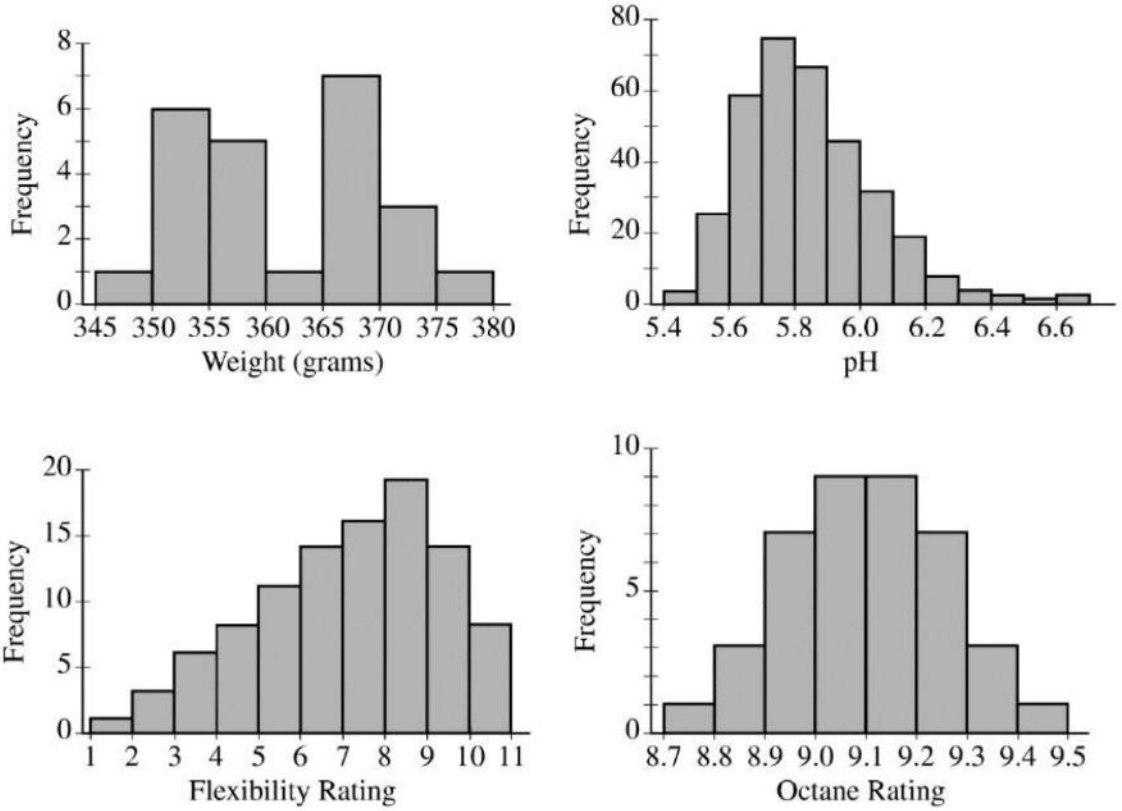
4. The distributions of four variables are shown in the following histograms.



Which of the following shapes is NOT represented by one of the four distributions?

(A) Uniform

(B) Bimodal

(C) Skewed to the left

(D) Skewed to the right

(E) Symmetric and unimodal

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5. A random sample of 1,092 people were asked whether color was a consideration in buying a new car. They were also asked to identify one additional feature that was important. The responses are shown in the table.

	Color Consideration			
	Yes	No	Maybe	Total
Comfort	40	96	12	148
Cost	108	68	8	184
Performance	62	62	12	136
Reliability	128	116	4	248
Safety	152	192	32	376
Total	490	534	68	1,092

Which of the following is closest to the proportion of people who responded no to color consideration and who identified safety as the additional feature that was important?

(A) 0.18

(B) 0.34

(C) 0.36

(D) 0.49

(E) 0.51

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6. Scientists estimate that the distribution of the life span of the Galápagos Islands giant tortoise is approximately normal with mean 100 years and standard deviation 15 years. Based on the estimate, which of the following is closest to the age of a Galápagos Islands giant tortoise at the 90th percentile of the distribution?

   (A) 80 years

   (B) 115 years

   (C) 120 years

   (D) 125 years

   (E) 130 years

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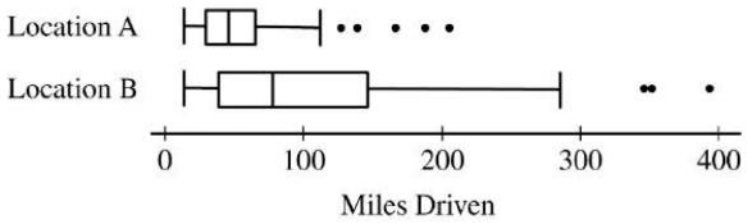
7. A car rental agency has two locations in a city. The boxplots below summarize the miles driven for one day of single-day car rentals at each location.



Based on the boxplots, which statement provides the best comparison of the two locations?

(A) The number of single-day rentals is greater for location A than for location B.

(B) The number of single-day rentals is less for location A than for location B.

(C) Compared with location A, the miles driven for location B display more variability, and the median is greater.

(D) Compared with location A, the miles driven for location B display less variability, and the median is greater.

(E) Compared with location A, the miles driven for location B display less variability, and the median is about the same.

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8. For the purpose of determining the value of its end-of-year inventory, a clothing store creates a list at the end of the year of every item currently in stock along with each item's wholesale price. Which of the following is the best description of the end-of-the-year activity?

(A) An experiment, because the items are treatments and wholesale prices are responses.

(B) An experiment, because the store does not know the total wholesale price of all the items.

(C) A sample survey, because the store wants to estimate the value of all items for the entire year.

(D) A sample survey, because the items currently in the store at the end of the year are a random sample of all items in the store for the entire year.

(E) A census, because the wholesale prices of all items are listed.

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9. A grocery store receives deliveries of corn from two farms, one in Iowa and the other in Ohio. Both farms produce ears of corn with mean weight 1.26 pounds. The standard deviation of the weights of the ears of corn from the farm in Ohio is 0.01 pound greater than that from the farm in Iowa. A randomly selected ear of corn from the farm in Iowa weighed 1.39 pounds, which has a standardized score of 1.645 for the distribution of weights for the Iowa corn. If an ear of corn from the farm in Ohio weighs 1.39 pounds, how many standard deviations from the mean is the weight with respect to the Ohio distribution?

(A) 1.46 standard deviations below the mean

(B) 1.46 standard deviations above the mean

(C) 1.65 standard deviations above the mean

(D) 1.88 standard deviations below the mean

(E) 1.88 standard deviations above the mean

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10. The distribution of number of hours worked by volunteers last year at a large hospital is approximately normal with mean 80 and standard deviation 7. Volunteers in the top 20 percent of hours worked will receive a certificate of merit. If a volunteer from last year is selected at random, which of the following is closest to the probability that the volunteer selected will receive a certificate of merit given that the number of hours the volunteer worked is less than 90 ?

(A) 0.077

(B) 0.123

(C) 0.134

(D) 0.618

(E) 0.923

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11. Resting heart rates, in beats per minute, were recorded for two samples of people. One sample was from people in the age-group of 20 years to 30 years, and the other sample was from people in the age-group of 40 years to 50 years. The five-number summaries are shown in the table.

Age-Group (years)	Minimum	Q1	Median	Q3	Maximum
20 to 30	60	71	72	75	84
40 to 50	60	70	73	76	85

The values of 60, 62, and 84 were common to both samples. The three values are identified as outliers with respect to the age-group 20 years to 30 years because they are either 1.5 times the interquartile range (IQR) greater than the upper quartile or 1.5 times the IQR less than the lower quartile.

Using the same method for identifying outliers, which of the three values are identified as outliers for the age-group 40 years to 50 years?

(A) None of the three values is identified as an outlier.

(B) Only 60 is identified as an outlier.

(C) Only 60 and 62 are identified as outliers.

(D) Only 60 and 84 are identified as outliers.

(E) The three values are all identified as outliers.

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12. As part of a demographic study, a college administrator needed to survey a sample of students from the college. From each major offered at the college, the administrator randomly selected 5 percent of the students with that major to participate in the survey. Which of the following is the best description of the type of sample selected by the administrator?

(A) Cluster sample

(B) Convenience sample

(C) Simple random sample

(D) Stratified random sample

(E) Systematic random sample

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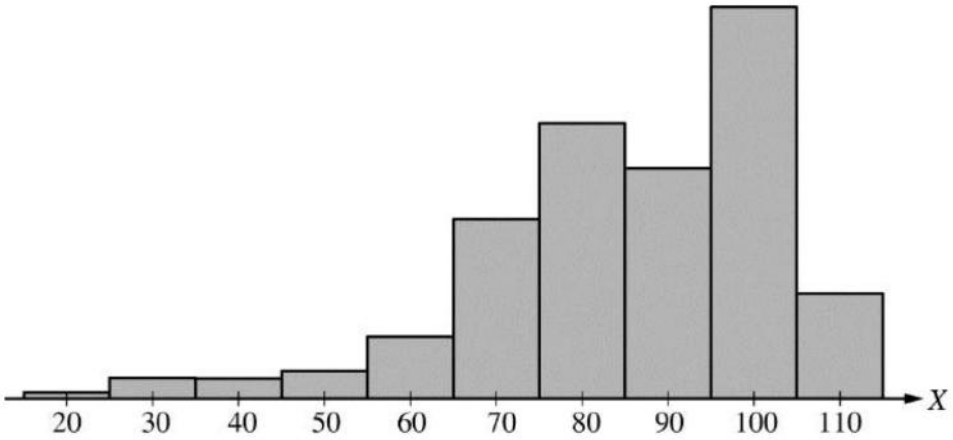
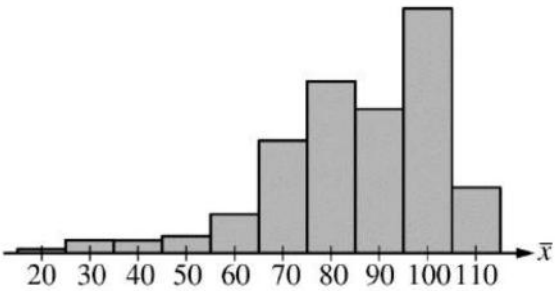
13. The graph shows the population distribution of random variable X with mean 85 and standard deviation 18.

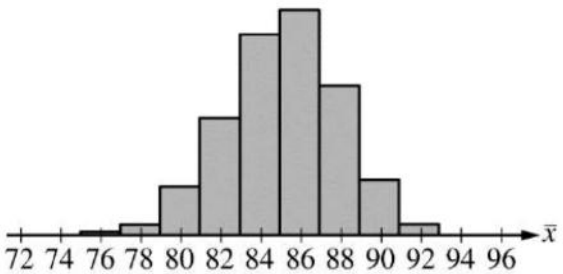


Which of the following graphs is a sampling distribution of the sample mean $\bar{x}$ for samples of size 40 taken from the population?

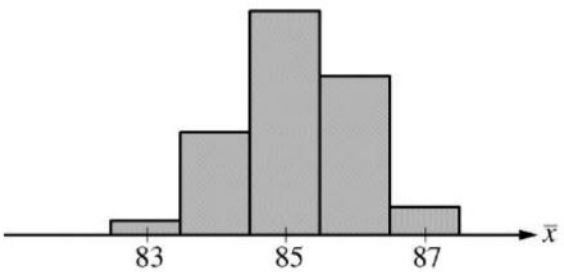
(A)



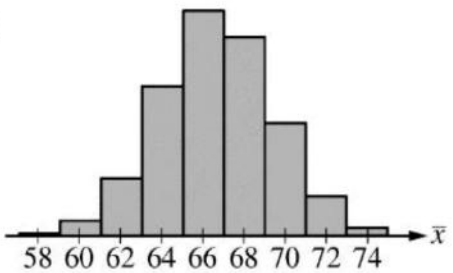
(B)



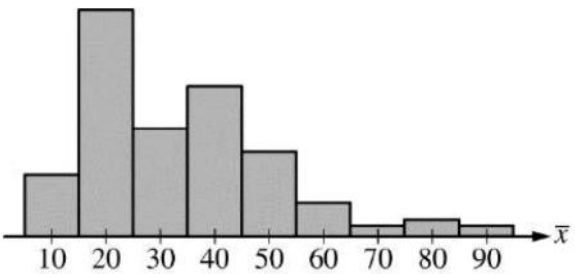
(C)



(D)



(E)



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14. A biologist studying trees constructed the confidence interval $(0.14, 0.20)$ to estimate the proportion of trees in a large forest that are dead but still standing. Using the same confidence level, the interval was later revised because the sample proportion had been miscalculated. The correct sample proportion was 0.27. Which of the following statements about the revised interval based on the correct sample proportion is true?

(A) The revised interval is narrower than the original interval because the correct sample proportion is farther from 0.5 than the miscalculated proportion is.

(B) The revised interval is narrower than the original interval because the correct sample proportion is closer to 0.5 than the miscalculated proportion is.

(C) The revised interval is wider than the original interval because the correct sample proportion is farther from 0.5 than the miscalculated proportion is.

(D) The revised interval is wider than the original interval because the correct sample proportion is closer to 0.5 than the miscalculated proportion is.

(E) The revised interval has the same width as the original interval.

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15. A research organization reported that 41 percent of adults who were asked to describe their day responded that they were having a good day rather than a typical day or a bad day. To investigate whether the percent would be different for high school students, 600 high school students were randomly selected. When asked to describe their day, 245 students reported that they were having a good day rather than a typical day or a bad day. Do the data provide convincing statistical evidence that the proportion of all high school students who would respond that they were having a good day is different from 0.41 ?

(A) No, because the p-value is less than any reasonable significance level.

(B) No, because the p-value is greater than any reasonable significance level.

(C) Yes, because the p-value is less than any reasonable significance level.

(D) Yes, because the p-value is greater than any reasonable significance level.

(E) Yes, because the expected value of the number of students who will report having a good day is 246, not 245.

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16. The director of a community recreation center conducted a six-week study to examine the effects of four types of exercise-strength training, flexibility training, aerobics, and jogging on maximal oxygen consumption. From the 40 members who participated, the director randomly assigned 10 members to each exercise type. Maximal oxygen consumption was measured for each member at the beginning of the study and again at the end of the six weeks. The director examined the change in maximal oxygen consumption for each member. Which of the following statements is a correct description of a feature of the study?

(A) The study has replication because there are four types of exercise.

(B) The study has replication because it was conducted over a six-week period.

(C) The response variable is the type of exercise with the greatest change in maximal oxygen consumption.

(D) The treatments in the study are strength training, flexibility training, aerobics, and jogging.

(E) The experimental units are the four different types of exercise.

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17. Sean and Evan are college roommates who have part-time jobs as servers in restaurants. The distribution of Sean's weekly income is approximately normal with mean $225 and standard deviation $25. The distribution of Evan's weekly income is approximately normal with mean $240 and standard deviation $15. Assuming their weekly incomes are independent of each other, which of the following is closest to the probability that Sean will have a greater income than Evan in a randomly selected week?

(A) 0.067

(B) 0.159

(C) 0.227

(D) 0.303

(E) 0.354

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18. According to data from the United States Elections Project, only 36 percent of eligible voters voted in the 2014 elections. For random samples of size 40, which of the following best describes the sampling distribution of $\hat{p}$, the sample proportion of people who voted in the 2014 elections?

(A) The sampling distribution is skewed to the left, with mean 0.36 and standard deviation 0.076.

(B) The sampling distribution is skewed to the right, with mean 0.64 and standard deviation 0.006.

(C) The sampling distribution is approximately normal, with mean 0.36 and standard deviation 0.076.

(D) The sampling distribution is approximately normal, with mean 0.36 and standard deviation 0.006.

(E) The sampling distribution is approximately normal, with mean 0.64 and standard deviation 0.076.

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19. To investigate whether the consumption of beetroot juice enhances exercise performance, a researcher selected a random sample of 50 student athletes from all the student athletes at a college. The athletes in the sample were randomly assigned to one of two groups. In one group, 25 athletes were given a daily dose of beetroot juice, and in the other group, the remaining athletes were given a daily dose of a placebo. At the end of six weeks of exercise training, the researcher compared the performances of the two groups. Based on the design of the investigation, which of the following is the largest population to which the results can be generalized?

(A) The 25 student athletes assigned to the beetroot juice group

(B) The 50 student athletes in the sample

(C) All student athletes at the college

(D) All students at the college

(E) All people who exercise

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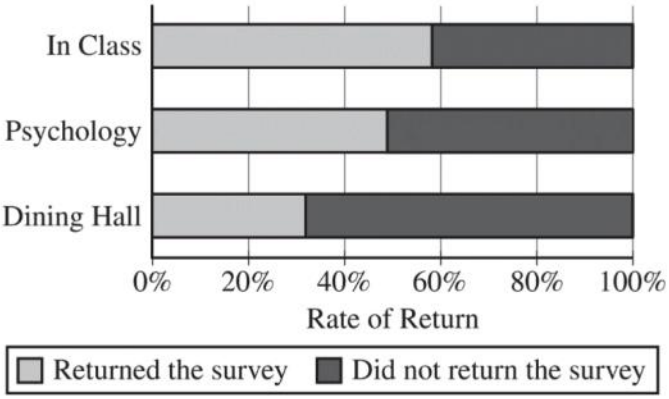
20. College researchers wanted to know under what conditions people are more likely to complete and return a survey. As part of a study, the researchers prepared three sets of identical surveys and used three methods of delivering and returning the surveys. The methods are described as follows.

- In Class: The surveys were given to students in a class, and students were asked to return completed surveys to their instructor.
- Psychology: The surveys were given to students participating in a psychology experiment, and students were asked to return completed surveys to a collection box in the hallway of the psychology building.
- Dining Hall: The surveys were given to students in the dining hall, and students were asked to return completed surveys to a collection box outside the dining hall.

The graph shows the percent of surveys returned and not returned for each delivery method.



Which statement about delivery method and rate of survey return is supported by the graph?

(A) There is a positive association between delivery method and rate of return.

(B) There is a negative association between delivery method and rate of return.

(C) The number of surveys given using the Dining Hall delivery method was less than the number given using either of the other delivery methods.

(D) The Psychology delivery method displays the most symmetric results; the other delivery methods display skewed results.

(E) The In Class delivery method had the greatest rate of return, and the Dining Hall delivery method had the least rate of return.

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21. The height and age of each child in a random sample of children was recorded. The value of the correlation coefficient between height and age for the children in the sample was 0.8. Based on the least-squares regression line created from the data to predict the height of a child based on age, which of the following is a correct statement?

(A) On average, the height of a child is 80% of the age of the child.

(B) The least-squares regression line of height versus age will have a slope of 0.8.

(C) The proportion of the variation in height that is explained by a regression on age is 0.64.

(D) The least-squares regression line will correctly predict height based on age 80% of the time.

(E) The least-squares regression line will correctly predict height based on age 64% of the time.

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22. At a certain restaurant, the distribution of wait times between ordering a meal and receiving the meal has mean 11.4 minutes and standard deviation 2.6 minutes. The restaurant manager wants to find the probability that the mean wait time will be greater than 12.0 minutes for a random sample of 84 customers. Assuming the wait times among customers are independent, which of the following describes the sampling distribution of the sample mean wait time for random samples of size 84 ?

(A) Approximately normal with mean 11.4 minutes and standard deviation 2.6 minutes

(B) Approximately normal with mean 11.4 minutes and standard deviation $\frac{2.6}{\sqrt{84}}$ minute

(C) Approximately normal with mean 12.0 minutes and standard deviation 2.6 minutes

(D) Binomial with mean $84(0.41)$ minutes and standard deviation $\sqrt{84(0.41)(0.59)}$ minutes

(E) Binomial with mean $84(0.5)$ minutes and standard deviation $\sqrt{84(0.5)(0.5)}$ minutes

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23. Two high schools have a similar number of students and parking lots of similar size. The safety officers at both schools want to investigate whether there is an average difference in the number of cars parked per day in the student parking lots for the school year. A random sample of 15 school days will be selected. For each selected day, the number of cars parked in the student parking lots will be counted at both schools and the difference will be recorded. Assuming all conditions for inference are met, which of the following is the appropriate test for the investigation?

(A) A two-sample z-test for a difference between proportions

(B) A two-sample t-test for a difference between means

(C) A matched-pairs t-test for a mean difference

(D) A chi-square test of homogeneity

(E) A chi-square test of independence

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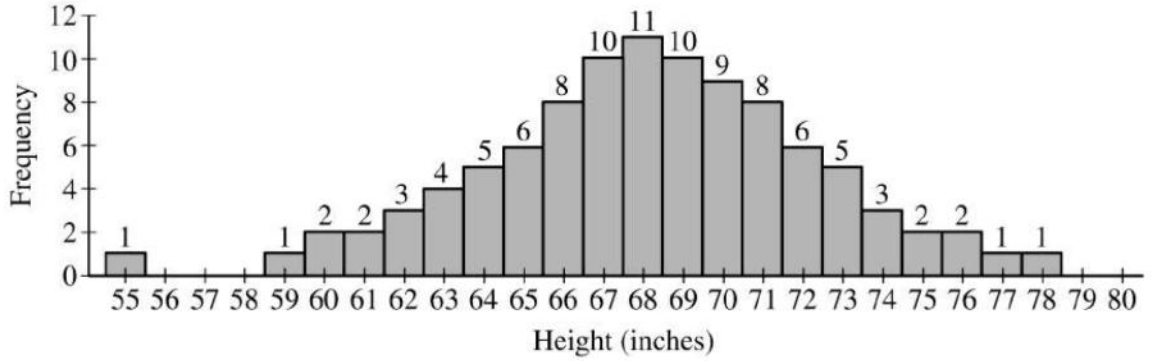
24. The histogram shows the distribution of heights, in inches, of 100 adult men.



Based on the histogram, which of the following is closest to the interquartile range, in inches, of the distribution?

(A) 2

(B) 5

(C) 9

(D) 12

(E) 15

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25. A state study on labor reported that one-third of full-time teachers in the state also worked part time at another job. For those teachers, the average number of hours worked per week at the part-time job was 13. After an increase in state teacher salaries, a random sample of 400 teachers who worked part time at another job was selected. The average number of hours worked per week at the part-time job for the teachers in the sample was 12.5 with standard deviation 6.5 hours. Is there convincing statistical evidence, at the level of $\alpha = 0.05$, that the average number of hours worked per week at part-time jobs decreased after the salary increase?

(A) No. The p-value of the appropriate test is greater than 0.05.

(B) No. The p-value of the appropriate test is less than 0.05.

(C) Yes. The p-value of the appropriate test is greater than 0.05.

(D) Yes. The p-value of the appropriate test is less than 0.05.

(E) Not enough information is given to determine whether there is convincing statistical evidence.

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26. A city department of transportation studied traffic congestion on a certain highway. To encourage carpooling, the department will recommend a carpool lane if the average number of people in passenger cars on the highway is less than 2. The probability distribution of the number of people in passenger cars on the highway is shown in the table.

Number of people	1	2	3	4	5
Probability	0.56	0.28	0.08	0.06	0.02

Based on the probability distribution, what is the mean number of people in passengers cars on the highway?

(A) 0.28

(B) 0.56

(C) 1.7

(D) 2

(E) 3

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27. To compare the effectiveness of two treatments, researchers conducted a well-designed experiment using a randomized block design in which the subjects were blocked by age-group (under 40 years and 40 years or older). Which of the following must be true about the randomized block design of the experiment?

(A) The number of subjects in each block is different.

(B) Treatments are randomly assigned to subjects within each block.

(C) The design cannot have a control group because subjects are blocked by age-group.

(D) The experiment uses a matched-pairs design, where subjects from one block are paired with subjects from the other block.

(E) The subjects in one block receive one treatment, and the subjects in the other block receive the other treatment.

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28. A die used in a certain board game has eight faces, of which 3 are red, 3 are yellow, and 2 are blue. Each face is equally likely to land faceup when the die is tossed. In the game, a player tosses the die until blue lands faceup, and the number of tosses before blue lands faceup is counted. For example, a player who tosses the sequence shown in the following table has tossed the die 3 times before blue lands faceup.

Toss number	1	2	3	4
Face color	yellow	yellow	red	blue

What is the probability that a player will toss the die <u>at least</u> 2 times before blue lands faceup?

(A) 0.1406

(B) 0.4219

(C) 0.4375

(D) 0.5625

(E) 0.5781

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29. A statistical test involves the following null and alternative hypotheses.

$$H_0: \mu = 64$$
$$H_a: \mu > 64$$

Which of the following describes a Type II error?

(A) Failing to reject the null hypothesis when the population mean is 64

(B) Failing to reject the null hypothesis when the population mean is greater than 64

(C) Rejecting the null hypothesis when the population mean is 64

(D) Rejecting the null hypothesis when the population mean is greater than 64

(E) Failing to reject the null hypothesis when the p-value is less than the significance level

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30. The marketing director for an ice cream company investigated whether there was a difference in preference for two new ice cream flavors—cotton candy and mango. Each participant from a large group of people was randomly assigned to taste one of the two flavors. After tasting, each person rated the flavor on a numerical scale from 1 to 5, where 1 represented strongly dislike and 5 represented strongly like. A two-sample t-interval for a difference between means (cotton candy minus mango) was constructed. Based on the interval, there was convincing statistical evidence of a difference in population mean flavor ratings, with mango having the greater sample mean rating. Which of the following could be the constructed interval?

(A) $(-20, -15)$

(B) $(-2.1, -1.3)$

(C) $(-1.4, 2.6)$

(D) $(1.5, 2.7)$

(E) $(15, 20)$

---

31. The director of a marketing department wants to estimate the proportion of people who purchase a certain product online. The director originally planned to obtain a random sample of 2,500 people who purchased the product. However, because of budget concerns, the sample size will be reduced to 1,500 people. Which of the following describes the effect of reducing the number of people in the sample?

(A) The variance of the sample will increase.

(B) The variance of the population will decrease.

(C) The variance of the sampling distribution of the estimator will increase.

(D) The variance of the sampling distribution of the estimator will decrease.

(E) The variance of the sampling distribution of the estimator will remain the same.

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32. From a random sample of 1,005 adults in the United States, it was found that 32 percent own an e-reader. Which of the following is the appropriate 90 percent confidence interval to estimate the proportion of all adults in the United States who own an e-reader?

(A) $0.32 \pm 1.960\left(\frac{(0.32)(0.68)}{\sqrt{1{,}005}}\right)$

(B) $0.32 \pm 1.645\left(\frac{(0.32)(0.68)}{\sqrt{1{,}005}}\right)$

(C) $0.32 \pm 2.575\sqrt{\frac{(0.32)(0.68)}{1{,}005}}$

(D) $0.32 \pm 1.960\sqrt{\frac{(0.32)(0.68)}{1{,}005}}$

(E) $0.32 \pm 1.645\sqrt{\frac{(0.32)(0.68)}{1{,}005}}$

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33. Measuring the height of a tree is usually more difficult than measuring the diameter of the tree. Therefore, many researchers use regression models to predict the height of a tree from its diameter measured at 4 feet 6 inches from the ground. The following computer output shows the results of a linear regression based on the heights, in feet, and the diameters, in inches, recorded from 31 felled trees.

	Estimate	Std Error	t value	Pr(> |t|)
Intercept	62.031	4.383	14.15	0.0000
Diameter	1.054	0.322	3.27	0.0028

Which of the following is a 95 percent confidence interval for the slope of the population regression line?

(A) $(0.001, 2.107)$

(B) $(0.396, 1.712)$

(C) $(0.423, 1.685)$

(D) $(0.732, 1.376)$

(E) $(53.07, 70.99)$

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34. A group of 80 people who had been diagnosed as prediabetic because of high blood glucose levels volunteered to participate in a study designed to investigate the use of cinnamon to reduce blood glucose to a normal level. Of the 80 people, 40 were randomly assigned to take a cinnamon tablet each day and the other 40 were assigned to take a placebo each day. The people did not know which tablet they were taking. Their blood glucose levels were measured at the end of one month. The results showed that 14 people in the cinnamon group and 10 people in the placebo group had normal blood glucose levels. For people similar to those in the study, do the data provide convincing statistical evidence that the proportion who would be classified as normal after one month of taking cinnamon is greater than the proportion who would be classified as normal after one month of not taking cinnamon?

(A) No conclusion can be made about the use of cinnamon because the people in the study were volunteers.

(B) There is convincing statistical evidence at the level of 0.01.

(C) There is convincing statistical evidence at the level of 0.05 but not at the level of 0.01.

(D) There is convincing statistical evidence at the level of 0.10 but not at the level of 0.05.

(E) There is not convincing statistical evidence at any reasonable significance level.

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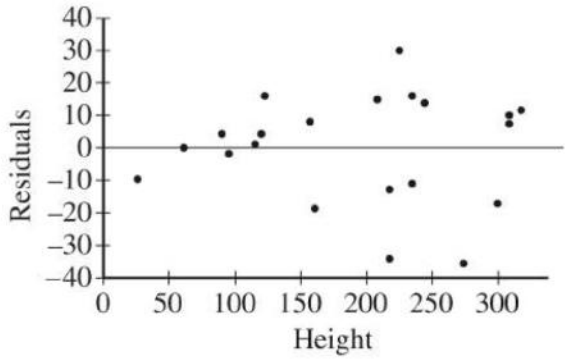
35. A student working on a physics project investigated the relationship between the speed and the height of roller coasters. The student collected data on the maximum speed, in miles per hour, and the maximum height, in feet, for a random sample of 21 roller coasters, with the intent of testing the slope of the linear relationship between maximum speed and maximum height. However, based on the residual plot shown, the conditions for such a test might not be met.



Based on the residual plot, which condition appears to have been violated?

(A) The errors are independent.

(B) The sum of the residuals is 0.

(C) The expected value of the errors is 0.

(D) There is a linear relationship between the response variable and the explanatory variable.

(E) The variance of the response variable is constant for all values of the explanatory variable.

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36. A researcher conducted a *t*-test of the hypotheses $H_0 : \mu = 38$ versus $H_a : \mu \neq 38$. The sample mean was 35 and the *p*-value for the test was 0.0627. What would the *p*-value have been if the researcher had used $H_a : \mu < 38$ as the alternative hypothesis?

(A) $1 - 0.0627$

(B) $1 - 2(0.0627)$

(C) $1 - \left(\frac{1}{2}\right)(0.0627)$

(D) $2(0.0627)$

(E) $\frac{1}{2}(0.0627)$

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37. High school students from track teams in the state participated in a training program to improve running times. Before the training, the mean running time for the students to run a mile was 402 seconds with standard deviation 40 seconds. After completing the program, the mean running time for the students to run a mile was 368 seconds with standard deviation 30 seconds. Let X represent the running time of a randomly selected student before training, and let Y represent the running time of the same student after training. Which of the following is true about the distribution of $X - Y$?

(A) The variables X and Y are independent; therefore, the mean is 34 seconds and the standard deviation is 10 seconds.

(B) The variables X and Y are independent; therefore, the mean is 34 seconds and the standard deviation is 50 seconds.

(C) The variables X and Y are not independent; therefore, the standard deviation is 50 seconds and the mean cannot be determined with the information given.

(D) The variables X and Y are not independent; therefore, the mean is 34 seconds and the standard deviation cannot be determined with the information given.

(E) The variables X and Y are not independent; therefore, neither the mean nor the standard deviation can be determined with the information given.

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38. A polling organization surveyed 2,002 randomly selected adults who are not scientists and 3,748 randomly selected adults who are scientists. Each adult was asked the question "Do you think that genetically modified foods are safe to eat?" Of those who are not scientists, 37 percent responded yes, and of those who are scientists, 88 percent responded yes. Which of the following is the standard error used to construct a confidence interval for the difference between the proportions of all adults who are not scientists and all adults who are scientists who would answer yes to the question?

(A) $\sqrt{\frac{(0.37)(0.63)}{2{,}002} + \frac{(0.88)(0.12)}{3{,}748}}$

(B) $\sqrt{\frac{(0.37)(0.63)}{2{,}002} - \frac{(0.88)(0.12)}{3{,}748}}$

(C) $\sqrt{\frac{(0.37)(0.63)}{2{,}002}} + \sqrt{\frac{(0.88)(0.12)}{3{,}748}}$

(D) $\sqrt{\frac{(0.70)(0.30)}{2{,}002}} + \sqrt{\frac{(0.70)(0.30)}{3{,}748}}$

(E) $\frac{(0.37)(0.63)}{\sqrt{2{,}002}} + \frac{(0.88)(0.12)}{\sqrt{3{,}748}}$

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39. A polling agency conducted a survey about social media in which each person in random samples of 1,000 men and 1,000 women was asked what factor he or she considers to be the most important when deciding whether to connect on social media with another person. The responses are shown in the table.

	Factor				
	Personal Friend	Stay in Touch	Mutual Friends	Business Networking	Other
Men	600	210	105	45	40
Women	650	224	65	15	46

What is the contribution to the chi-square test statistic for men who selected business networking as the most important factor?

(A) 0.5

(B) 5

(C) 7.5

(D) 30

(E) 45

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40. A national survey asked 1,501 randomly selected employed adults how many hours they work per week. Based on the collected data, a 95 percent confidence interval for the mean number of hours worked per week for all employed adults was given as $(41.18, 42.63)$. Which of the following statements is a correct interpretation of the interval?

(A) Ninety-five percent of all employed adults work between 41.18 hours and 42.63 hours per week.

(B) The probability is 0.95 that a sample of size 1,501 will produce a mean between 41.18 hours and 42.63 hours.

(C) Of all samples of size 1,501 taken from the population, 95% of the samples will have a mean between 41.18 hours and 42.63 hours.

(D) We are 95% confident that the mean number of hours worked per week for employed adults in the sample is between 41.18 hours and 42.63 hours.

(E) We are 95% confident that the mean number of hours worked per week for all employed adults is between 41.18 hours and 42.63 hours.

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

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Table entry for z is the probability lying below z.

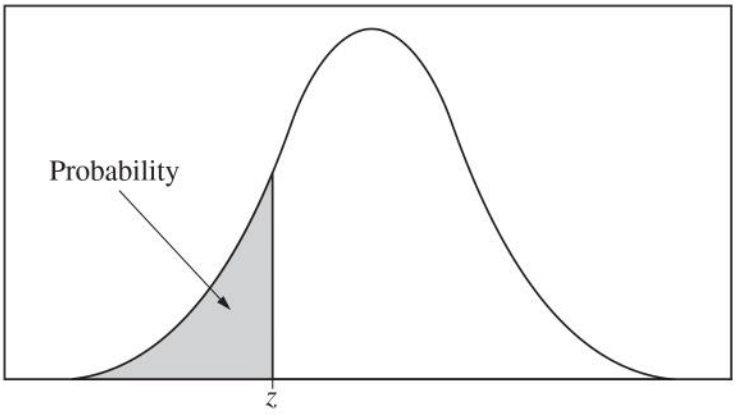

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

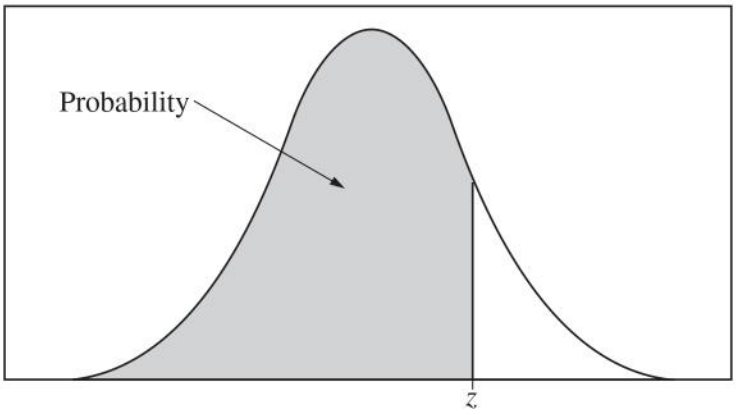

Table A (*Continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

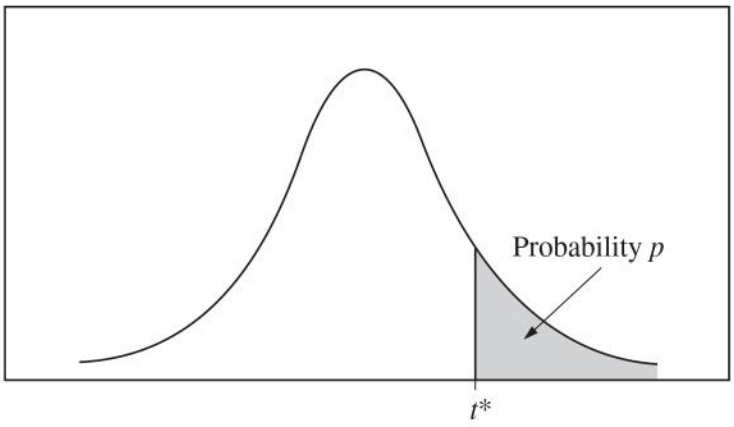

Table B t **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

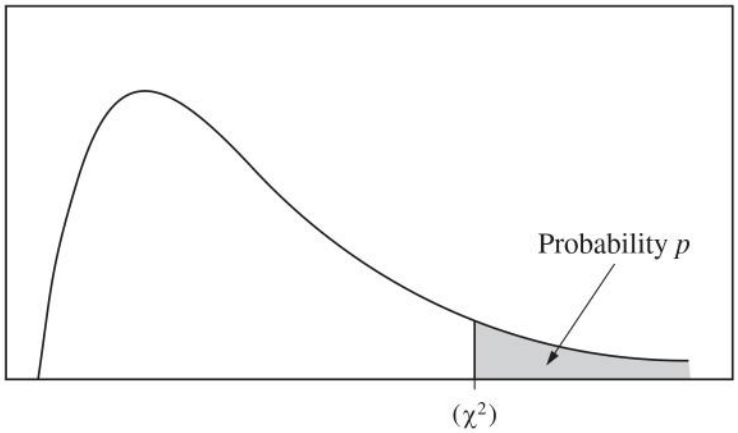

Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

AP® Statistics Exam

SECTION II: Free Response

2019

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Electronic Device
Graphing calculator expected

Part A

Number of Questions
5
Suggested Time
1 hour and 5 minutes
Percent of Section II Score
75%

Part B

Number of Questions
1
Suggested Time
25 minutes
Percent of Section II Score
25%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐
4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I
Form Code 4PBP4-S

90

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\Sigma\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\Sigma\left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_X = \sum x_i p_i$$

$$\mathrm{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_X = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If i is the mean of a random sample of size n from an infinite population with mean u and standard deviation o, then:

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic $\pm$ (critical value) $\bullet$ (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

STATISTICS
SECTION II
Part A
Questions 1-5
Spend about 1 hour and 5 minutes on this part of the exam.
Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Thirty-four college students were asked how much money they spent on textbooks for the current semester. Their responses are shown in the following stemplot.

1	2 3 3 4 5 5 6 7 8
2	1 2 3 4 5 6 8 8 9 9 9
3	1 2 2 7 8 9
4	1 4 5 7
5	1 3
6	2
7	
8	1

Key: 1|2 = $120

(a) Describe a procedure for identifying potential outliers, and use the procedure to decide whether there are outliers among the responses for the money spent on textbooks.

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GO ON TO THE NEXT PAGE.

(b) Based on the stemplot, write a few sentences describing the distribution of money spent on textbooks for the 34 students.

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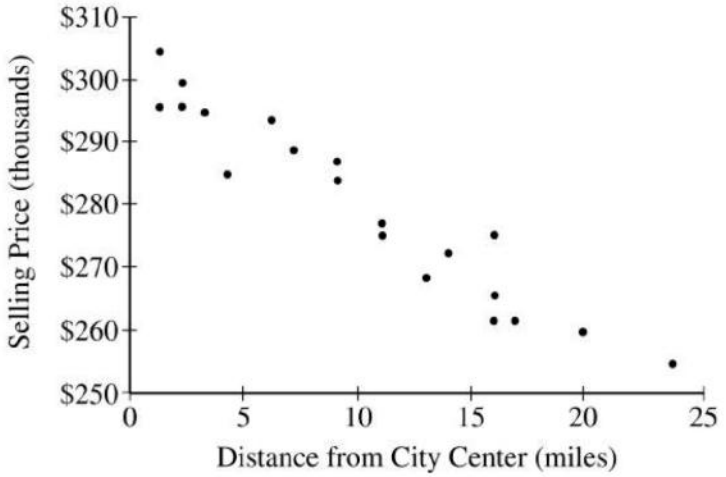
2. A real estate agent working in a large city believes that, for three-bedroom houses, the selling price of the house decreases by approximately $2,000 for every mile increase in the distance of the house from the city center. To investigate the belief, the agent obtained a random sample of 20 three-bedroom houses that sold in the last year. The selling price, in thousands of dollars, and the distance from the city center, in miles, for each of the 20 houses are shown in the scatterplot. The table shows computer output from a regression analysis of the data.



Predictor	Coef	SE Coef	T	P
Constant	301.7	1.80	167.17	0.000
Distance	–2.158	0.149	–14.45	0.000
	S = 4.4336		R-sq = 92.1%	

(a) Assume all conditions for inference are met. Construct and interpret a 95 percent confidence interval for the slope of the least-squares regression line.

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(b) Does the confidence interval contradict the agent's belief about the relationship between selling price and distance from the city center? Justify your answer.

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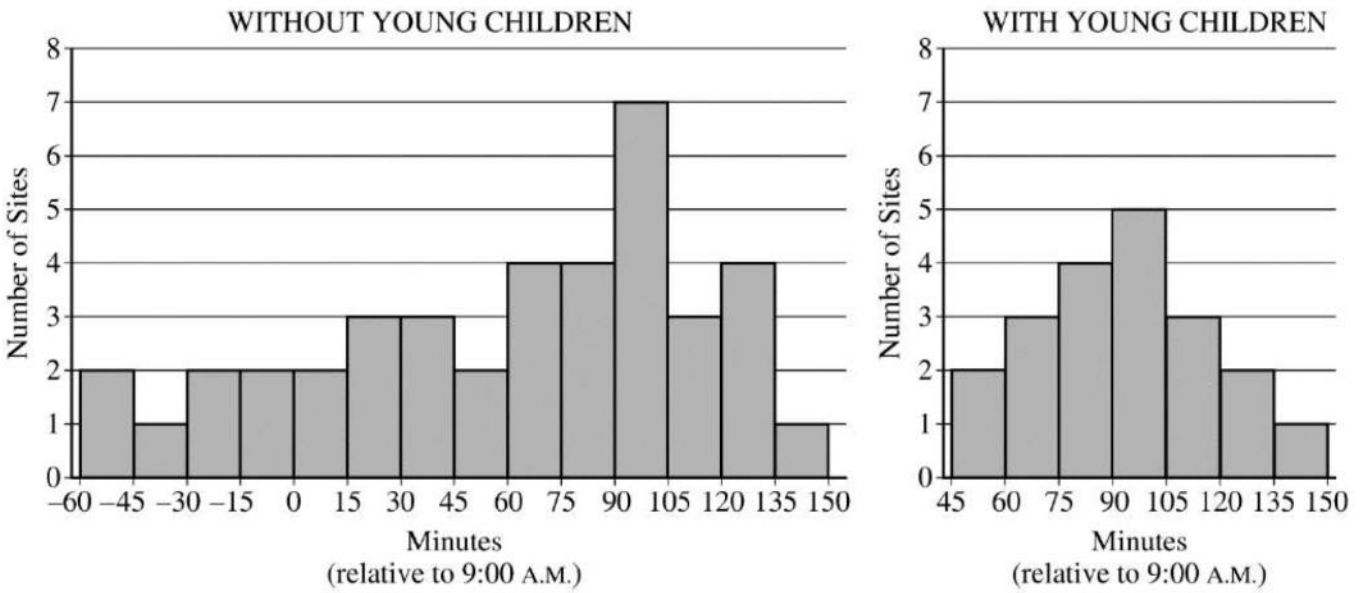
3. River Run campground has sites for people to use for camping. The sites can be reserved for a certain number of days. To help with cleaning and maintenance, the campground requests an exit time (the time at which campers leave the site) of 9 A.M. on the last day of the reservation.

To estimate the typical exit time, the manager of River Run selected a random sample of 60 sites. Of the selected sites, 40 were reserved by people without young children, and 20 were reserved by people with young children. The following histograms summarize the exit times, recorded as minutes relative to 9 A.M. For example, an exit time of 9:30 A.M. is 30 minutes relative to an exit time of 9 A.M. Each interval contains possible values from the left endpoint up to but not including the right endpoint.



(a) Consider the two histograms.

(i) How many of the 60 sites had an exit time before 8:30 A.M.?

(ii) How many of the 60 sites had an exit time of 11:00 A.M. or later?

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(b) Compare the distributions of the exit times for those without young children and those with young children.

(c) Based on the histograms, what is a reasonable estimate of the median exit time for the random sample of 60 sites? Explain your reasoning.

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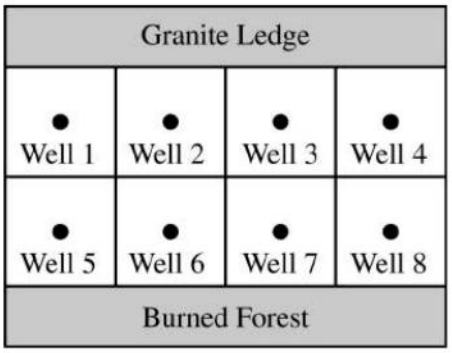
4. Arsenic is a naturally occurring chemical that can enter groundwater through eroding granite or from a burned forest. A health organization recommends drinking water should contain no more than 10 parts per billion (ppb) of arsenic. A company produces filters to clean arsenic from private wells that could be affected by the groundwater.

The company wants to investigate the effectiveness of a new filter compared to that of an older filter. They will test the filters on a field that is bordered on one side by a granite ledge and on the other side by a burned forest. The field is divided into 8 square plots of equal size, and a well to collect groundwater is drilled in the center of each plot. One filter will be used in each well. The following diagram shows the placement of the wells in the field.



The company will use four of each type of filter to conduct the investigation. A randomized block design will be used.

(a) Assuming there is a difference in the effectiveness of the two filters, under what conditions will a randomized block design be better for detecting the difference than a completely randomized design?

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(b) Identify the wells, by number, that will be included in each block.

(c) Describe how to assign filters to wells to create a randomized block design.

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5. For each day that Sasha travels to work, the probability that she will experience a delay due to traffic is 0.2. Each day can be considered independent of the other days.

   (a) For the next 21 days that Sasha travels to work, what is the probability that Sasha will experience a delay due to traffic on at least 3 of the days?

   (b) What is the probability that Sasha's first delay due to traffic will occur after the fifth day of travel to work?

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(c) Consider a random sample of 21 days that Sasha will travel to work. For the proportion of those days that she will experience a delay due to traffic, is the sampling distribution of the sample proportion approximately normal? Justify your answer.

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STATISTICS

SECTION II

Part B

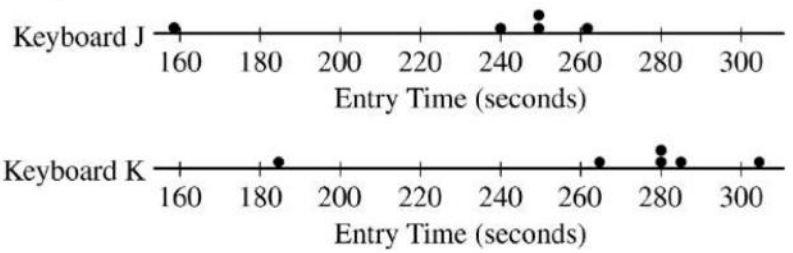
Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Researchers are studying two different designs of computer keyboards, J and K, to investigate the effectiveness of the design on the speed of data entry. The researchers believe there is a tendency for people entering data with keyboard J to have faster entry times compared with people entering data with keyboard K. Using the same set of data for entry, the researchers randomly assigned 5 people to keyboard J and 6 people to keyboard K and recorded the number of seconds each person took to enter the data. The following dotplots show the observed entry times for the two keyboards.



(a) Explain why it is not appropriate to conduct a two-sample t-test for the difference in population means.

(b) Based on the dotplots, explain why it might be more appropriate to compare population medians instead of population means.

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One test used to compare population medians is the Wilcoxon Rank Sum Test. Under the assumption that the shape and variability of the distributions are the same, the test uses the rankings of the combined observed values. To conduct the test, the entry times for keyboards J and K are combined into one group and then ranked from 1 to n_T, the total number of observed values in the combined group. The observed entry times, in seconds, for both keyboard types are shown in the following table.

	Observed Entry Times					
J	158	240	248	251	261	
K	184	267	279	280	284	305

(c) Consider the observed entry times for keyboards J and K.

(i) Complete the following table to assign ranks to the observed entry times for keyboards J and K combined.

Rank	1	2	3	4	5	6	7	8	9	10	11
Keyboard	J	K	J							K	K
Time	158	184	240							284	305

(ii) Use the completed table in (i) to calculate the sum of the ranks assigned to each keyboard.

Sum of ranks for J (SR_J):

Sum of ranks for K (SR_K):

The hypotheses for the Wilcoxon Rank Sum Test are as follows.

H_0 : The median of the distribution of entry times for all users of keyboard J and the median of the distribution of entry times for all users of keyboard K are the same.

H_a : The median of the distribution of entry times for all users of keyboard J is less than the median of the distribution of entry times for all users of keyboard K.

The test statistic W for the test is $W = SR_J - \frac{n_J(n_J + 1)}{2}$, where n_J is the number of observations for keyboard J.

(d) Use the formula to calculate the test statistic W for the keyboard data.

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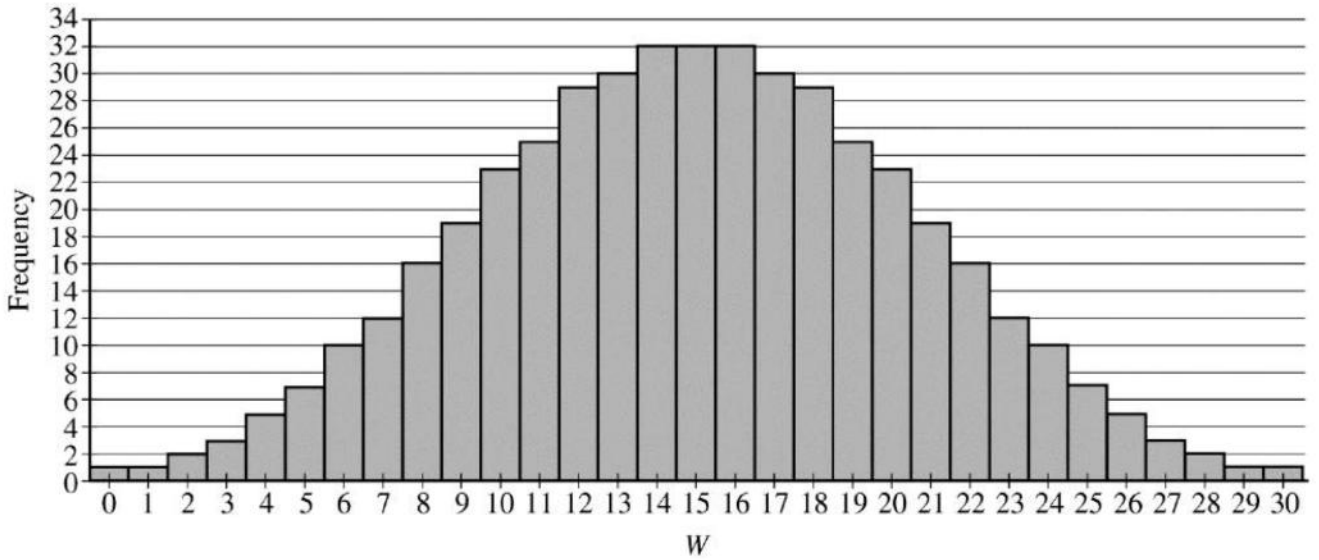
There are 462 possible assignments of 11 ranks to 5 Js and 6 Ks. If the null hypothesis is true, the 462 assignments are equally likely. The following graph shows the sampling distribution of all possible values of W resulting from the 462 assignments.



(e) The least possible value of W in the sampling distribution is 0.

(i) Find the value of SR_J for $W = 0$.

(ii) Assign 5 ranks to keyboard J and 6 ranks to keyboard K so that $W = 0$. Show your assignments by completing the following table.

Rank	1	2	3	4	5	6	7	8	9	10	11
Keyboard											

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(f) Use the test statistic from part (d) and the graph of the sampling distribution to decide whether there is convincing statistical evidence, at the level of $\alpha = 0.05$, that the median of the distribution for all users of keyboard J is less than the median of the distribution for all users of keyboard K. Support your answer.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE FRONT COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

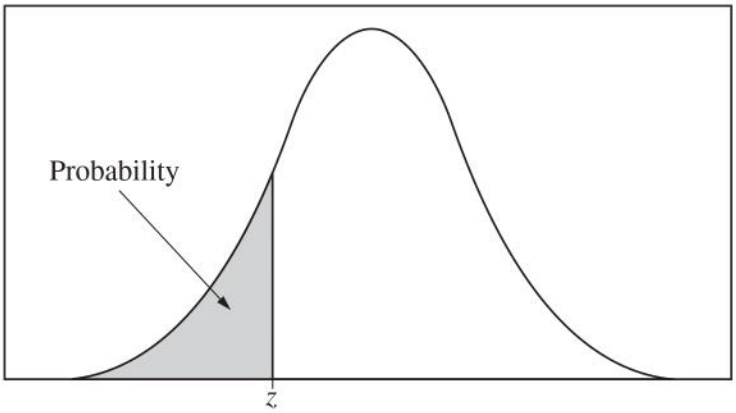

Table entry for z is the probability lying below z.

Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z.

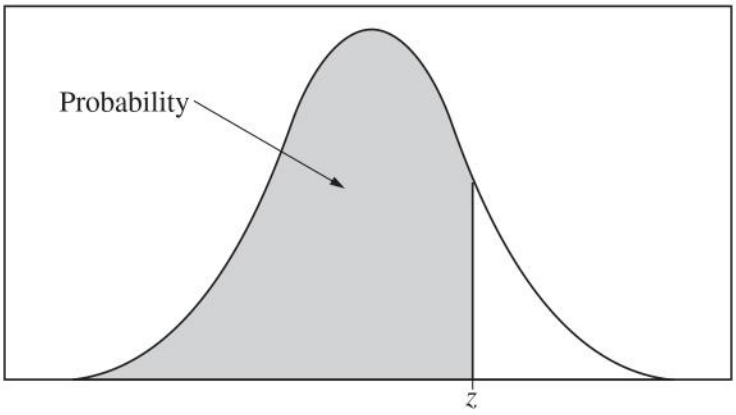

Table A (*Continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

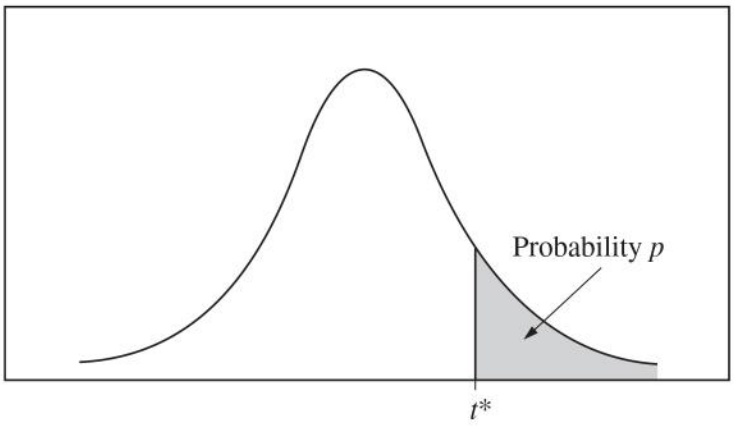

Table B t **distribution critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the point (χ^2) with probability p lying above it.

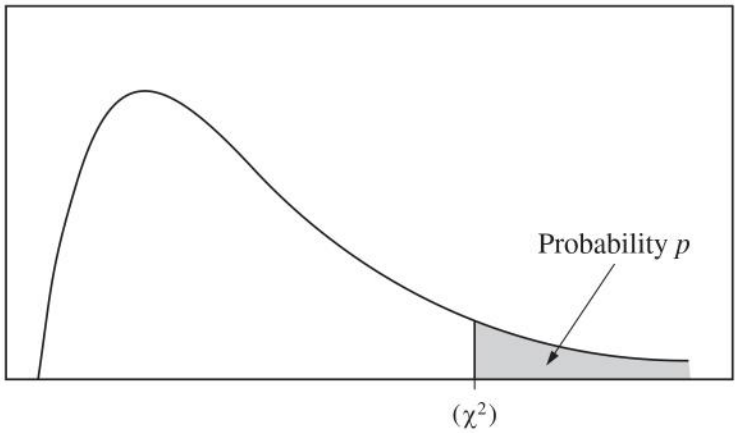

Table C χ^2 **critical values**

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2