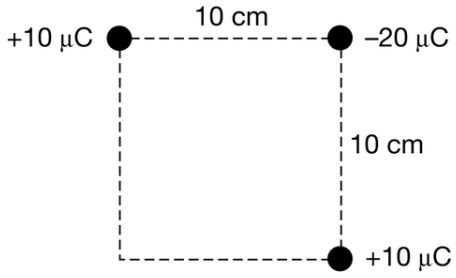
**PHYSICS C: ELECTRICITY AND MAGNETISM**
**SECTION I**
**Time—45 minutes**
**35 Questions**

**Directions:** Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.

**Note:** To simplify calculations, you may use $g$ = 10 m/s$^2$ in all problems.

**Questions 1-2**



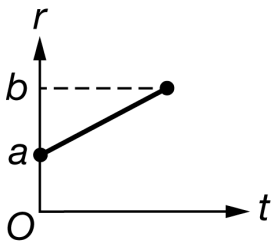

Two particles, each with charge $+10\mu$C, are fixed in place at opposite corners of a square, and a particle with charge $-20\mu$C is fixed in place at one of the other corners, as shown in the figure above. The length of each side of the square is 10 cm.

1. A particle with charge $-5\mu$C is placed in the exact center of the square and then released. In what direction, if any, will it initially move?

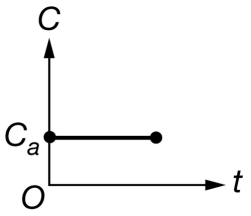
   (A) Toward the top of the page.
   (B) Toward the bottom of the page.
   (C) Toward the corner of the square with the $-20\mu$C particle.
   (D) Toward the empty corner of the square.
   (E) In no direction, because it will not move.

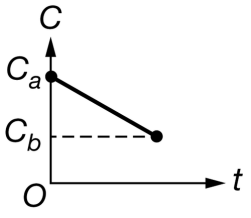
2. The $-5\mu$C particle is now removed and a different particle is placed at the remaining corner of the square. After those changes, the electric potential at the center of the square is zero. What is the charge on the new particle?

   (A) $-20\mu$C
   (B) $-10\mu$C
   (C) Zero
   (D) $+10\mu$C
   (E) $+20\mu$C




3. An isolated metallic sphere of radius $a$ has a capacitance $C_a$. Suppose the radius of the sphere could be increased over time to a new radius $b$, as shown in the figure above. Which of the following would be the graph of the sphere's capacitance $C$ as a function of time $t$?

(A)



(B)
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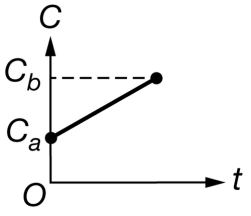
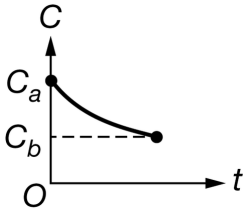



(D)
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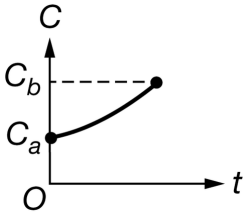
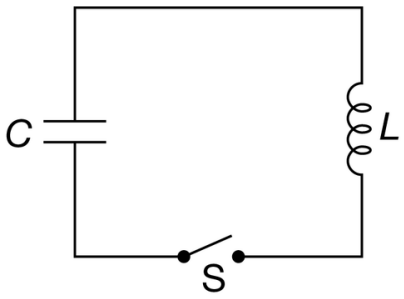
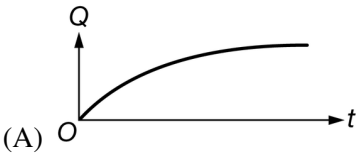



4. In the circuit shown above, the capacitor is initially charged. Which of the following graphs best represents the charge $Q$ on the capacitor versus time $t$ after switch is closed at time $t = 0$?

(A)
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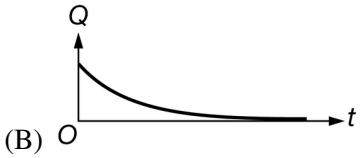


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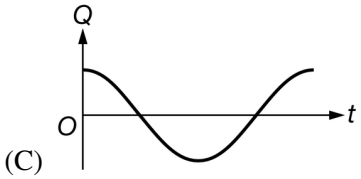


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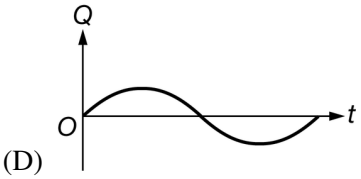
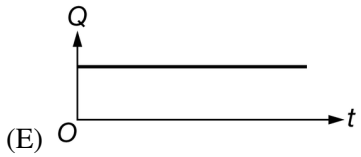
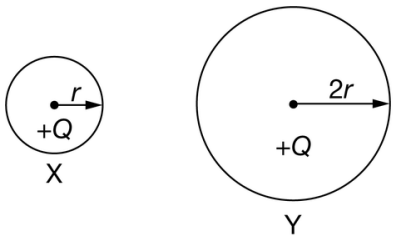



(E)


**Questions 5-6**




Note: Figure not drawn to scale.

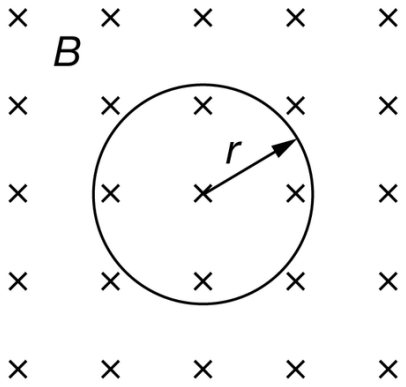
The two conducting spheres shown above each initially have a net positive charge $+Q$. Sphere $X$ has radius $r$ and sphere $Y$ has radius $2r$. The two spheres are held fixed at a separation much greater than their radii. A conducting wire is now used to connect the two spheres. After any resulting current flow has ceased, the wire is removed.

5. The net charge on sphere $X$ after the wire is removed is

   (A) $+\frac{1}{3}Q$
   (B) $+\frac{1}{2}Q$
   (C) $+\frac{2}{3}Q$
   (D) $+Q$
   (E) $+\frac{3}{2}Q$

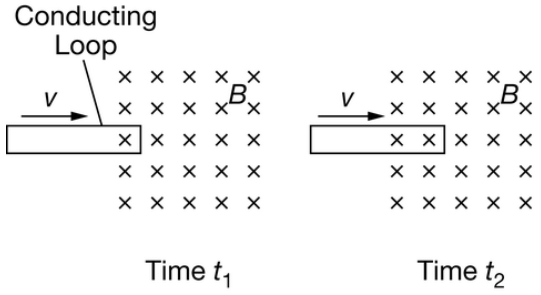
6. What is the ratio $V_X/V_Y$ of the electric potential at the surface of sphere $X$ to the electric potential at the surface of sphere $Y$ after the wire is removed?

   (A) $1/4$
   (B) $1/2$
   (C) $1/1$
   (D) $2/1$
   (E) $4/1$



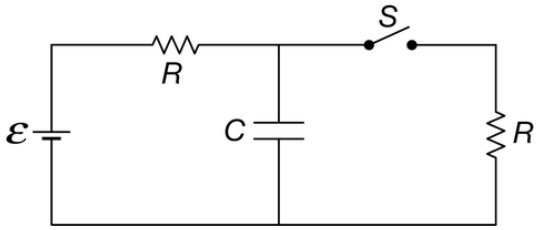

7. A single loop of wire is at rest in a magnetic field that is directed into the page, as shown in the figure above. The loop has a radius of $r$ and a resistance of $R$. The magnetic field has a magnitude $B$ that changes as a function of time $t$ according to the equation $B = \alpha t + \beta t^2$, where $\alpha$ and $\beta$ are both positive constants in units of T/s and T/s$^2$, respectively. Which of the following expressions gives the current in the loop as a function of time $t$?

   (A) $\frac{\beta t \pi r^2}{R}$
   (B) $\frac{(\alpha + 2\beta t)\pi r^2}{R}$
   (C) $\frac{(\alpha + 2\beta t) t \pi r^2}{R}$
   (D) $\frac{(\beta t) 2\pi r}{R}$
   (E) $\frac{(\alpha + 2\beta t) 2\pi r}{R}$




8. A rectangular conducting loop is moving at constant speed $v$ into a uniform magnetic field of magnitude $B$ that is directed into the page. At the two times $t_1$ and $t_2$ shown in the figures, the currents in the loop are $I_1$ and $I_2$, respectively. Which of the following correctly compares currents $I_1$ and and $I_2$ provides evidence?

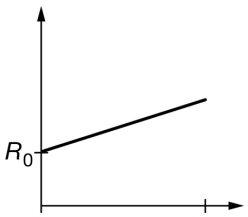
   (A) $I_2 < I_1$, because more of the loop is in the field at time $t_2$.
   (B) $I_2 > I_1$, because it takes more force to move the loop at constant speed at time $t_2$.
   (C) $I_2 = I_1$, because the current in the loop is zero at both times.
   (D) $I_2 = I_1$, because the loop experiences the same rate of change in flux at both times.
   (E) $I_2 > I_1$, because more of the loop is in the field at time $t_2$.

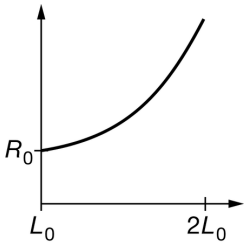



9. In the circuit shown above, switch $S$ is left open for a very long period of time and is then closed. What is the initial current that passes through $S$ immediately after it is closed?

   (A) 0
   (B) $\frac{\varepsilon}{2R}$
   (C) $\frac{\varepsilon}{R}$
   (D) $\frac{2\varepsilon}{R}$
   (E) $\frac{\varepsilon}{RC}$

10. A cylindrical resistor of resistance $R_0$ and length $L_0$ is created using a total volume $V_0$ of a flexible material with constant resistivity. The resistor is carefully stretched so that its volume does not change, it remains a cylinder, and its length doubles. Which of the following graphs best shows the resistance of the wire as a function of the length of the wire as the wire is stretched?

(A)



(B)
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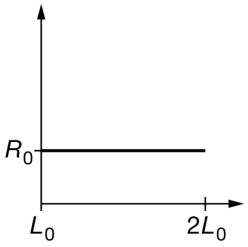


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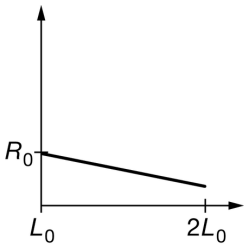
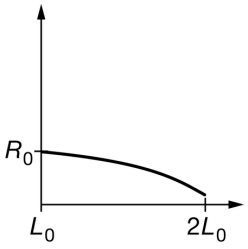
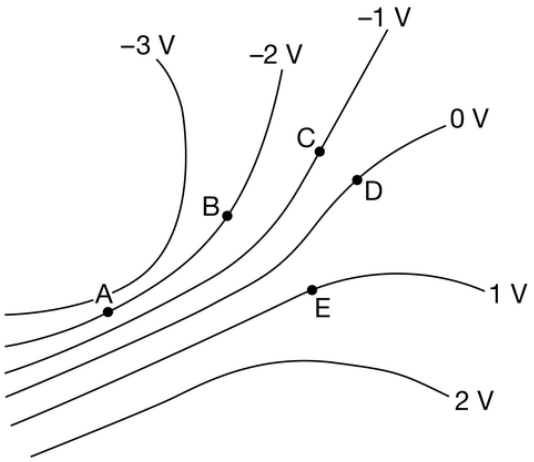



(E)


**Questions 11-13**




The diagram above shows equipotential lines in a particular region.

11. Which of the following vectors, if any, best shows the direction of the electric field at point D?

(A) 

(B) 

(C) 

(D) 

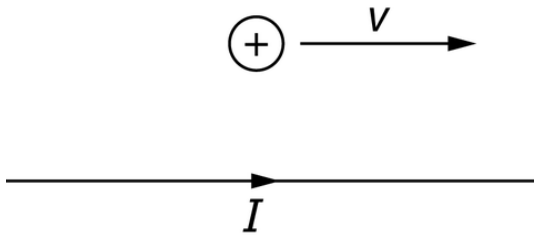
(E) None of the above; the magnitude of the field is zero.

12. At which of the following points is the magnitude of the electric field greatest?

(A) A
(B) B
(C) C
(D) D
(E) E

13. What is the work done by the electric force in moving an electron from point B to point A?

(A) $-2.0$ J
(B) 0 J
(C) $3.2 \times 10^{-19}$ J
(D) 2.0 J
(E) It cannot be determined because it depends on the path taken from B to A.




14. A positively charged particle is moving with speed $v$ to the right in the plane of the page, as shown. The wire carries a current $I$ directed toward the right, as shown. Which of the following indicates the direction of the magnetic force on the particle due to the current?

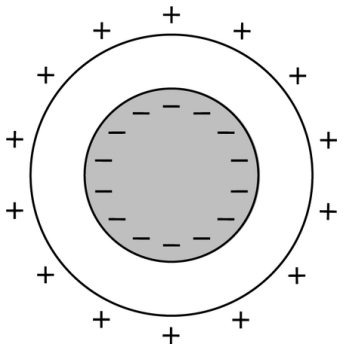
(A) 

(B) 

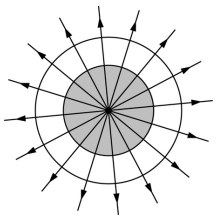
(C) 

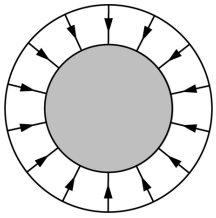
(D) 

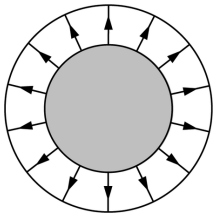
(E) The direction is undefined, because no magnetic force is exerted on the moving particle.

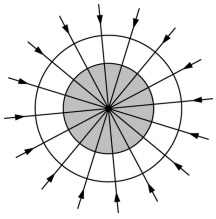


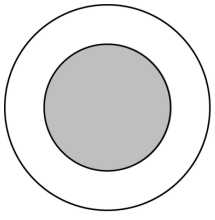
15. A spherical capacitor is constructed with a spherical conductor surrounded by a thin spherical conducting shell. The inner sphere carries a charge $-Q$ and the outer shell carries a charge $Q$, as shown in the figure above. Which of the following correctly shows the electric field lines for the capacitor?

(A)


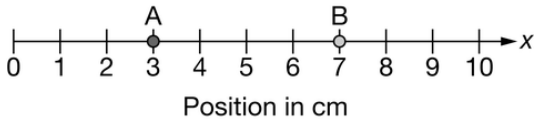
(B)


(C)


(D)


(E) ($E = 0$ everywhere)


**Questions 16-17**




Two small spheres are positioned a distance of 4 cm from each other on the $x$-axis as shown above. Sphere $A$ has a charge of $+3.0\mu$C, while sphere $B$ has a charge of $-2.0\mu$C.

16. The electric potential at $x = 6$ cm is most nearly

(A) $V = +9.0 \times 10^5$ V
(B) $V = +2.1 \times 10^6$ V
(C) $V = -9.0 \times 10^5$ V
(D) $V = -2.1 \times 10^6$ V
(E) $V = -7.0 \times 10^7$ V

17. What is the $x$-coordinate of the point at which the net electric field due to the spheres is zero?

(A) 1.8 cm
(B) 4.6 cm
(C) 5.4 cm
(D) 17.8 cm
(E) 24.8 cm

18. A steadily increasing current in a resistor of resistance $R$ is given as function of time $t$ by $I(t) = bt$, where $b$ is a constant. What is the total energy dissipated in the resistor during the time interval $0 \leq t \leq T$?

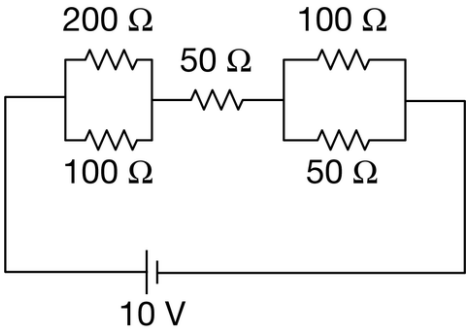
(A) $2b^2TR$

(B) $\frac{1}{2}b^2T^2R$

(C) $b^2T^2R$

(D) $\frac{1}{3}b^2T^3R$

(E) $b^2T^3R$




19. The figure above shows five resistors connected to an ideal battery. The equivalent resistance of the circuit is most nearly

(A) 500 Ω

(B) 350 Ω

(C) 200 Ω

(D) 150 Ω

(E) 50 Ω

20. A student is constructing a capacitor using two parallel plates and a dielectric. The student intends for the capacitance to be 1.0 F. Once the capacitor is connected to a power supply and the charge is measured, the calculated value of the capacitance is only 0.8 F. Which of the following is a possible reason for the discrepancy?

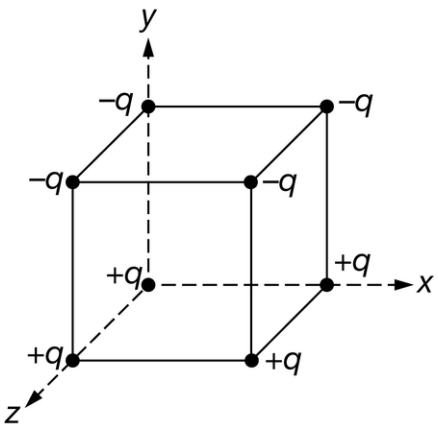
(A) The potential difference of the power supply is half what was expected.

(B) The dielectric constant was larger than expected.

(C) The separation of the plates was greater than what was expected.

(D) The area of the plates was larger than the student had anticipated.

(E) The wires used to connect the capacitor had significant resistance.




21. Charges of equal magnitude are placed at the corners of a cube, with charges of $-q$ at the top and charges of $+q$ at the bottom, as shown in the figure above. If a positive charge is placed at the center of the cube, what is the direction of the force on it due to the other charges?

(A) $+x$

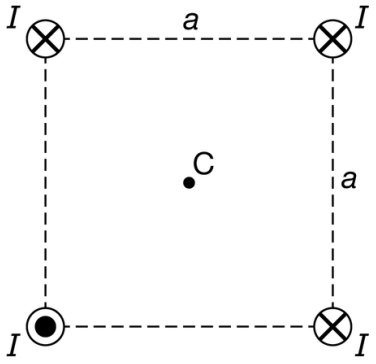
(B) $+y$

(C) $-y$

(D) $+z$

(E) $-z$

**Questions 22-23**




Four very long wires are at the corners of a square and each carry current $I$ perpendicular to the plane and in the directions shown. The sides of the square are length $a$. Point $C$ is at the center of the square.

22. Which of the following is a correct expression for the magnitude of the net force per length on the top right wire due to the other three wires?

(A) 0

(B) $\dfrac{\sqrt{2}I^2\mu_0}{4\pi a}$

(C) $\dfrac{I^2\mu_0}{2\pi a}$

(D) $\dfrac{\sqrt{2}I^2\mu_0}{2\pi a}$

(E) $\dfrac{\sqrt[3]{2}I^2\mu_0}{4\pi a}$

23. If the magnetic field at point $C$ is 2.0 $\mu$T and $a = 0.10$ m, then $I$ is most nearly

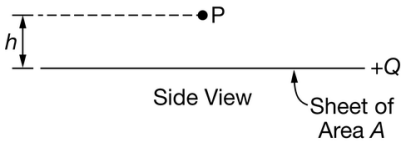
(A) 0.22 A

(B) 0.35 A

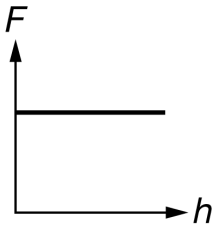
(C) 0.45 A

(D) 0.64 A

(E) 1.3 A




24. A large, horizontal, thin sheet of positive charge of area $A$ has uniform charge density and carries a total charge $+Q$. Point $P$ is a short distance $h$ above the sheet, as shown above. Which of the following is a correct graph for the magnitude of the force $F$ exerted on a proton placed at point $P$ as a function of $h$?

(A)
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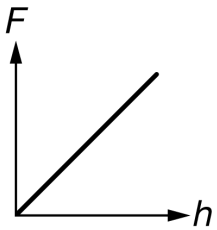


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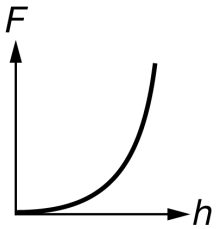
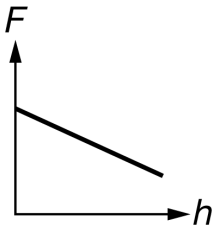



(D)
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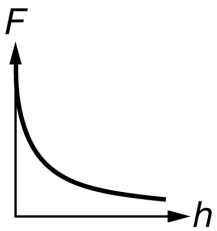
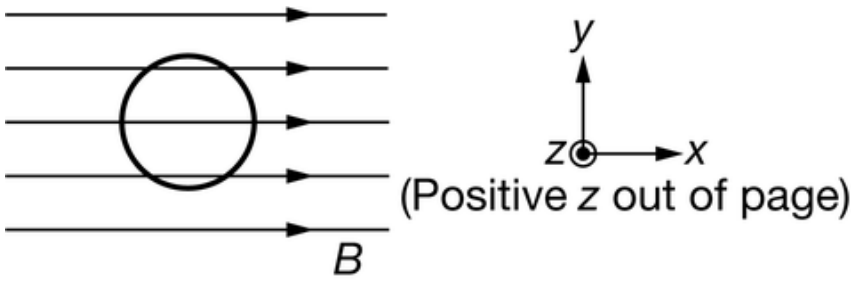





25. A circular loop of wire in the plane of the page is in a uniform magnetic field of magnitude $B$ directed to the right, as shown in the figure above. The magnetic field extends above and below the page. For which of the following would there be an induced current in the loop?

I. Rotate the loop around the $x$-axis.

II. Rotate the loop around the $y$-axis.

III. Rotate the loop around the $z$-axis.

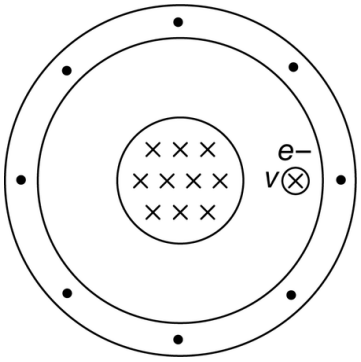
(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III




26. A coaxial cable consists of a cylindrical wire inside of a cylindrical shell of wire. The inner wire has a current $I$ directed into the page, and the outer wire has a current $I$ out of the page. An electron has a velocity directed into the page, as shown in the figure above. What is the direction of the magnetic force exerted on the electron?

(A) Toward the left side of the page.

(B) Toward the right side of the page.

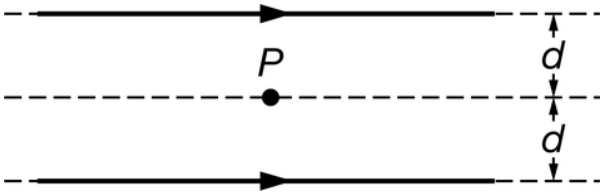
(C) Toward the bottom of the page.

(D) There is no magnetic force on the electron because the magnetic field in the region between the inner and outer wires is parallel to the velocity.

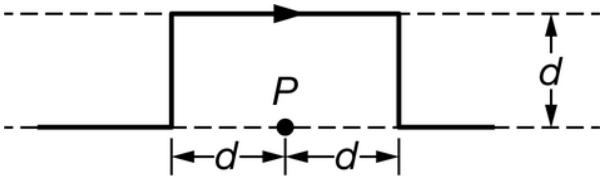
(E) There is no magnetic force on the electron because there is no magnetic field in the region between the inner and the outer wires.

27. Wires are shown in five different configurations below. Option *A* shows two wires, and all other options show a single wire. For all the configurations, the wires have a current *I*. For each configuration, a labeled point *P* is the same distance *d* from the indicated wires. Which of the following configurations creates a magnetic field of the greatest magnitude at point *P*?

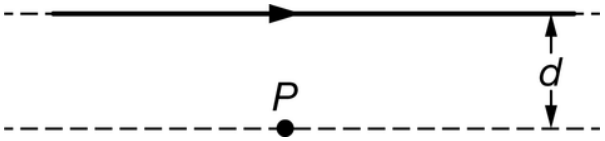
(A)



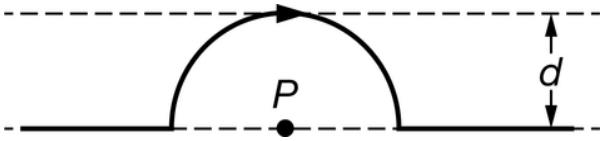

(B)



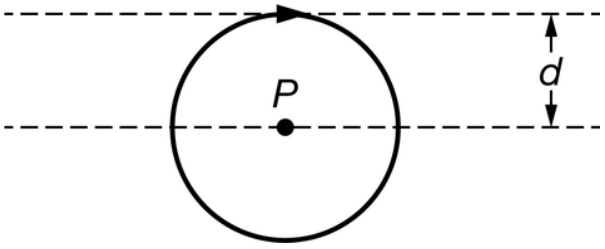

(C)

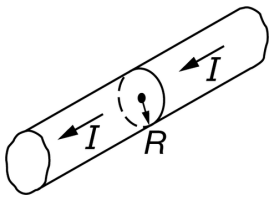



(D)



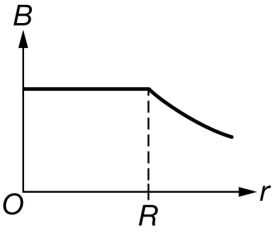

(E)



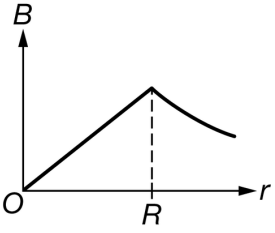




28. The long wire of radius *R* shown above carries a current *I* that is uniformly distributed over its cross-sectional area. Which of the following graphs best represents the magnitude of the magnetic field *B* as a function of distance *r* from the axis of the wire?

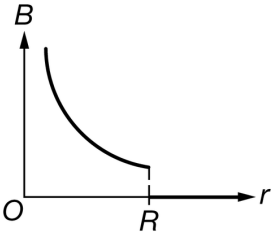
(A)



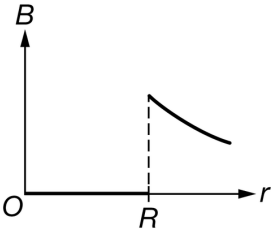

(B)



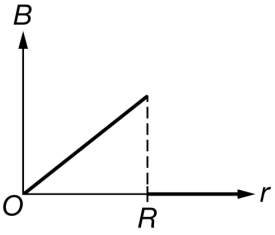

(C)

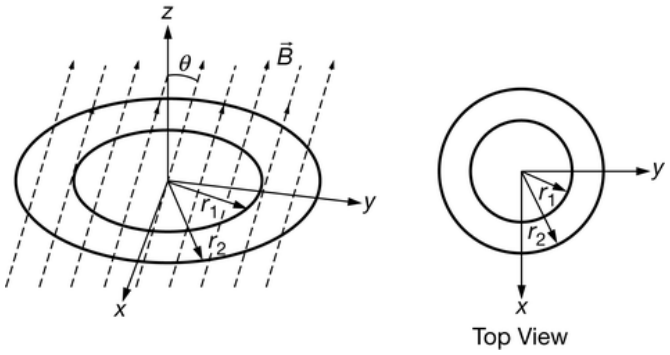



(D)




(E)






29. Two circular loops of thin metal wire of radius $r_1$ and radius $r_2$ are positioned on the horizontal $xy$-plane. A magnetic field of magnitude $B$ is directed upward through the loops at an angle $\theta$ with the vertical $z$-axis, as shown in the figure above. The magnetic flux in the space between the loops is

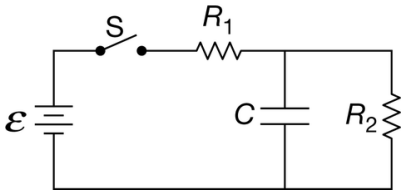
(A) $2\pi B \sin\theta (r_2 - r_1)$

(B) $2\pi B \cos\theta (r_2 - r_1)$

(C) $\pi B \cos\theta (r_2 - r_1)^2$

(D) $\pi B \sin\theta (r_2^2 - r_1^2)$

(E) $\pi B \cos\theta (r_2^2 - r_1^2)$




30. A circuit contains an ideal battery with potential difference $\varepsilon$, two resistors with resistances $R_1$ and $R_2$, a capacitor of capacitance $C$, and switch $S$, as shown above. The switch is closed. A long time after the switch is closed, the positive plate of the capacitor has charge $+Q$. What change to the circuit can a student make to increase the amount of charge stored on the positive plate of the capacitor?

(A) Replace $R_1$ with a new resistor of larger resistance.

(B) Replace $R_2$ with a new resistor of larger resistance.

(C) Replace the capacitor with a capacitor that is identical except that there is more separation between its plates.

(D) Replace the capacitor with a capacitor that is identical except that the plates have a smaller area.

(E) Replace the battery with an ideal battery that has a lower potential difference.

31. Isolated conducting spheres 1 and 2 have radii $R_1$ and $R_2$, respectively. The spheres are separated by a large distance, and spheres 1 and 2 have charges of $+Q_1$ and $+Q_2$, respectively. The electric field at the surface of sphere 1 is $e_1$, and the electric field at the surface of sphere 2 is $E_2$. The ratio $E_1/E_2$ is

(A) $\dfrac{Q_1 R_2}{Q_2 R_1}$

(B) $\dfrac{Q_1 R_1}{Q_2 R_2}$

(C) $\dfrac{Q_1 R_2^2}{Q_2 R_1^2}$

(D) $\dfrac{Q_1 R_1^2}{Q_2 R_2^2}$

(E) $\dfrac{Q_2 R_1^2}{Q_1 R_2^2}$

32. An electric heater and a lamp are connected in parallel across a 120 V circuit. The heater operates at 1000 W and the lamp operates at 100 W. The ratio of the resistance of the heater to the resistance of the lamp is

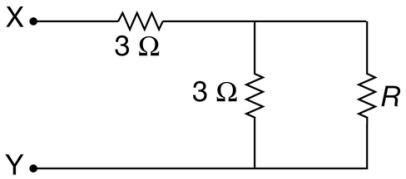
(A) $\frac{1}{100}$

(B) $\frac{1}{10}$

(C) 1

(D) 10

(E) 100




33. The diagram above shows a segment of a circuit for which the equivalent resistance between points $X$ and $Y$ is 5 Ω. What is the value of resistor $R$?

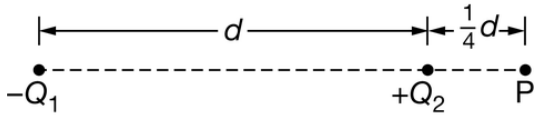
(A) 1 Ω

(B) 2 Ω

(C) 3 Ω

(D) 5 Ω

(E) 6 Ω




34. Two stationary point charges $-Q_1$ and $+Q_2$ are separated by a distance $d$ as shown above. If the distance from $+Q_2$ to point is $\frac{1}{4}d$, what must be the relationship between the magnitudes of the charges for the electric field to be zero at point $P$?

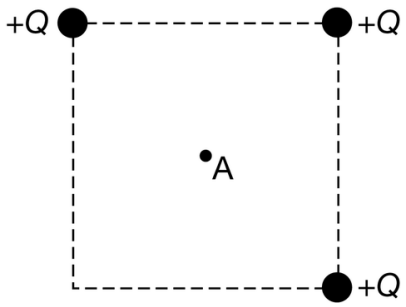
(A) $Q_1 = 4Q_2$

(B) $Q_1 = 5Q_2$

(C) $Q_1 = 9Q_2$

(D) $Q_1 = 16Q_2$

(E) $Q_1 = 25Q_2$




35. Three particles of equal charge $+Q$ are arranged at 3 corners of a square, as shown above. Point $A$ is located at the center of the square. What charge could be placed at the empty corner to make the electric potential at point $A$ zero?

(A) $+3Q$

(B) $+Q$

(C) Zero

(D) $-Q$

(E) $-3Q$

**STOP**
**END OF ELECTRICITY AND MAGNETISM SECTION I**

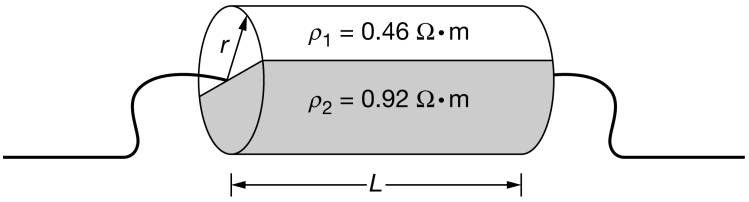
**PHYSICS C: ELECTRICITY AND MAGNETISM**

**SECTION II**

**Time—45 minutes**

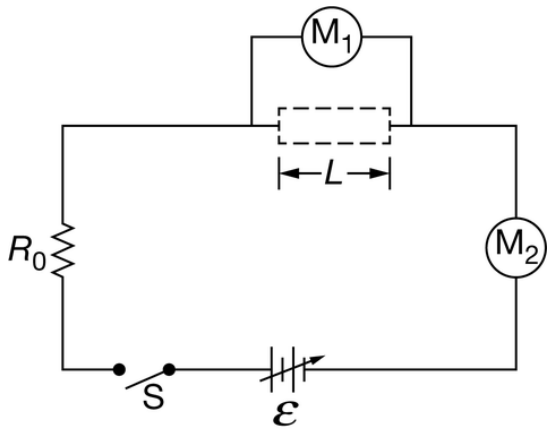
**3 Questions**

**Directions:** Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.




1. A cylinder of length $L = 0.25$ m and radius $r = 0.020$ m is made of two different materials, as shown. The top half has a resistivity of $\rho_1 = 0.46\ \Omega$·m, and the bottom half has a resistivity of $\rho_1 = 0.92\ \Omega$·m. The cylinder has connection wires at the center of each end, as shown.

   (a) Calculate the resistance of this cylinder.




(b) The cylinder from part (a) is placed into the circuit shown above in which the dashed rectangular area represents the cylinder. The rest of the circuit consists of a variable voltage source with potential difference $\varepsilon$, ideal meters $M_1$ and $M_2$, switch $S$, and a resistor of known resistance $R_0$. The switch is closed, and a current $I$ is measured for the circuit.

i. Which of the following indicates the measurement made by meter $M_1$? Check all that apply.

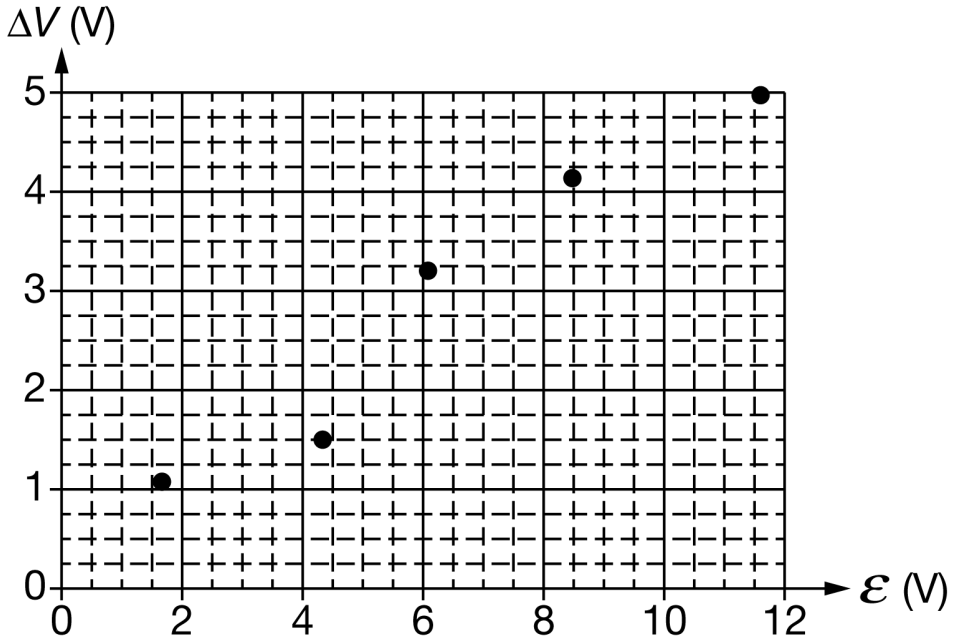
_____ Potential difference across the cylinder _____ Current through the cylinder
_____ Potential difference across resistor $R_0$ _____ Current through resistor $R_0$
_____ Potential difference of the voltage source _____ Current through the voltage source

ii. Which of the following indicates the measurement made by meter $M_2$? Check all that apply.

_____ Potential difference across the cylinder _____ Current through the cylinder
_____ Potential difference across resistor _____ Current through resistor
_____ Potential difference of the voltage source _____ Current through the voltage source

(c) Derive an expression for the potential difference $\Delta V$ across the cylinder once the switch is closed. Express your answer in terms of $\varepsilon$, $R_0$, the cylinder's resistance $R_C$, and physical constants, as appropriate.

(d) Students run an experiment with the circuit in which they measure the potential difference $\Delta V$ across the cylinder for several different potential differences of the variable voltage source. The data are plotted on the graph below.




i. Draw a best-fit line to represent the data.

ii. Use your best-fit line to calculate the resistance $R_0$.

(e) i. Identify one possible source of error in the experiment and explain how it would affect the measurement of the potential difference $\Delta V$ across the cylinder.

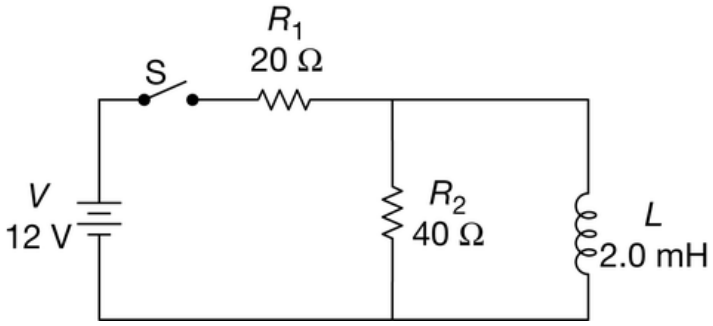
ii. While conducting the experiment, students notice that the measurement reading on meter $M_2$ depends on whether meter $M_1$ is in the circuit. Which of the following relationships is correct about the measurement reading on $M_2$?

_____ It is greater with meter $M_1$ in the circuit.

_____ It is less with meter $M_1$ in the circuit.

Explain why the reading on meter $M_2$ changes when meter $M_1$ is connected to the circuit.

iii. Explain how you would change the procedure to determine an experimental value for the resistance of meter $M_2$.




2. The circuit shown above consists of an ideal battery, two resistors, an inductor, and a switch. The switch is initially open. At time $t = 0$, the switch is closed.

(a) i. Calculate the current in resistor $R_1$ immediately after the switch is closed.

ii. Calculate the potential difference across the inductor immediately after the switch is closed.

iii. On the axes below, sketch the potential difference $\Delta V$ across the inductor as a function of time $t$.



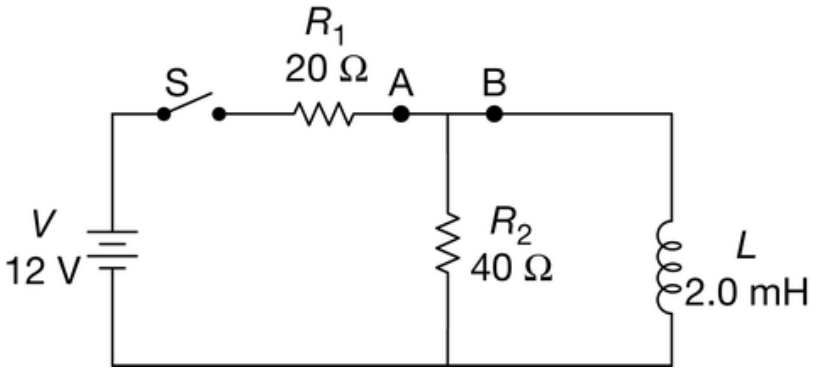
(b) After a long time, the switch is now opened.

i. Calculate the current in resistor immediately after the switch is opened.

ii. Is the current up through $R_2$, down through $R_2$, or zero?

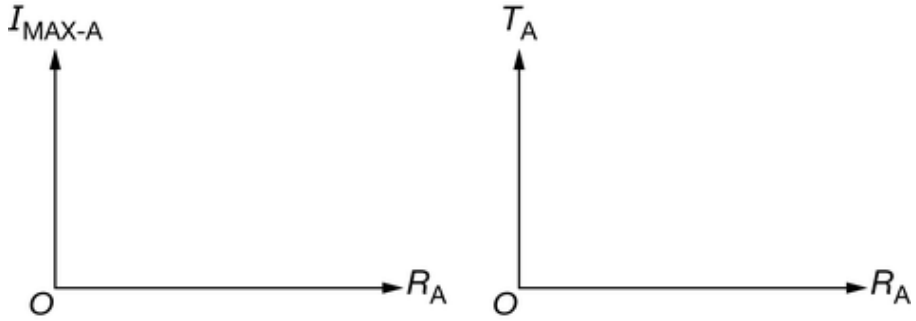
(c) Write, but do NOT solve, the differential equation for the current $I(t)$ as a function of $t$ through the inductor after the switch is opened.

(d) Using integral calculus, derive an equation to show that the equation of the current as a function of time is $T(t) = I_0 e^{-t/\tau}$, where $\tau = \dfrac{L}{R_2}$.

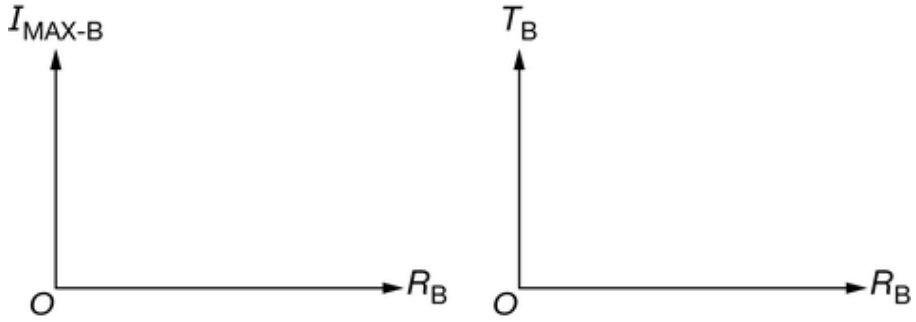


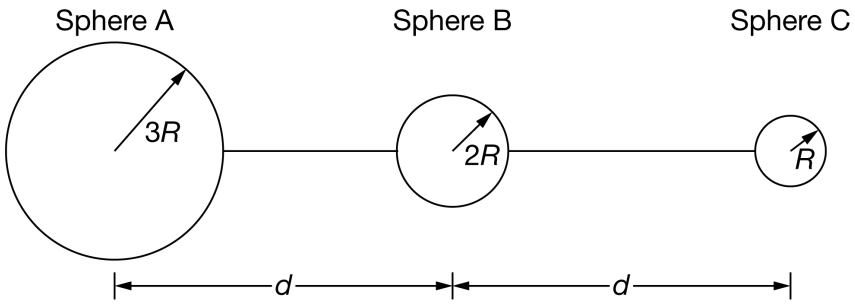

(e) The procedure is repeated, with the switch closed for a long time and then opened. Points $A$ and $B$ are indicated in the circuit diagram, as shown.

i. Resistors of different resistances $R_A$ are placed at position $A$. The switch is closed for a long time and then opened. The following two measurements are made for each resistor: the maximum current $I_{\text{MAX-A}}$ in the inductor immediately after the switch is opened and the time $T_A$ it takes the current in the inductor to reach 67% of its initial value after the switch is opened. On the axes below, sketch graphs of $I_{\text{MAX-A}}$ and $T_A$ as functions of $R_A$.




ii. Resistors of different resistances $R_B$ are now placed at position $B$. The switch is closed for a long time and then opened. The following two measurements are made for each resistor: the maximum current $I_{\text{MAX-B}}$ in the inductor immediately after the switch is opened and the time $T_A$ it takes the current in the inductor to reach 67% of its initial value after the switch is opened. On the axes below, sketch graphs of $I_{\text{MAX-B}}$ and $T_B$ as functions of $R_B$.






Note: Figures not drawn to scale.

3. Three solid conducting spheres are connected with two very long, thin conducting wires, as shown. They are all initially neutral; then a charge of $12Q$ is transferred to the leftmost sphere. Enough time elapses that the spheres achieve electrostatic equilibrium. Assume $d >> R$ and that the system of spheres and wires is isolated from any external forces.

   (a) Calculate the charge on each sphere. Express your answers in terms of $Q$.

   (b) Rank the spheres according to their surface charge density, with 1 indicating the greatest surface charge density. If two spheres have the same value, then give them the same ranking.
   _____ Sphere $A$ _____ Sphere $B$ _____ Sphere $C$

   (c) The magnitudes of the electric field at the surfaces of spheres $A$ and $C$ are $E_A$ and $E_C$, respectively. Calculate the ratio $E_A/E_C$.

(d) Is the tension in the left wire greater than, less than, or equal to the tension in the right wire? Assume nonelectric forces are negligible.
_____ Greater than _____ Less than _____ Equal to
Justify your answer.

The wire between spheres $B$ and $C$ breaks. Assume the force of gravity is negligible.

(e) Determine the new charges on spheres $A$ and $B$. Express your answer in terms of $Q$.
Justify your answer.

(f) After the wire breaks, sphere $C$ accelerates away from the other two spheres.

i. On the dot below that represents sphere $C$, draw an arrow that represents the direction of the acceleration of sphere $C$ immediately after the wire breaks. The arrow must start on, and point away from, the dot.



ii. Derive an expression for the kinetic energy of the system of three spheres when sphere $C$ is very far away from spheres $A$ and $B$. Express your answer in terms of $q$, $R$, $d$, and physical constants, as appropriate.

**STOP**
**END OF EXAM**